

Progress In Electromagnetics Research Symposium (PIERS)  
[2P\_13 SC3 Optical Microcavities Aug. 9, 2016]

# The effect of Raman scattering in Kerr comb generation in a Silica Toroidal Microcavity

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# Outline

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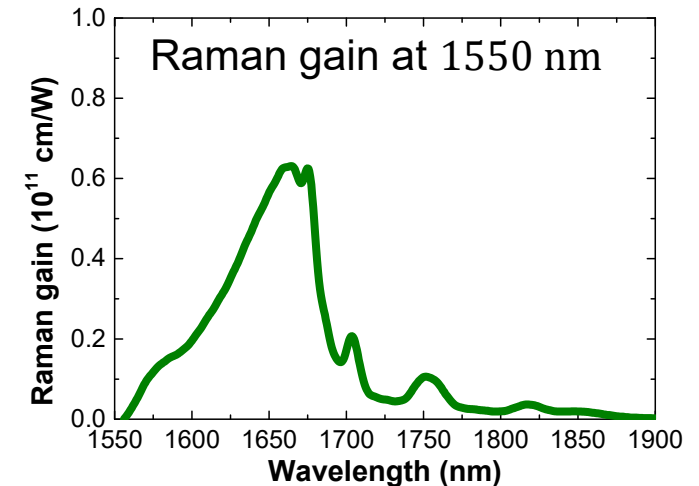
- Background & Motivation
  - Raman scattering in Kerr comb generation
- Numerical modeling
  - Simultaneous Lugiato-Lefever Equations
- Numerical results
  - Dependence on Q factor
  - Raman soliton generation
- Summary



# Raman lasing in high-Q silica microcavity

- Stimulated Raman Scattering(SRS)

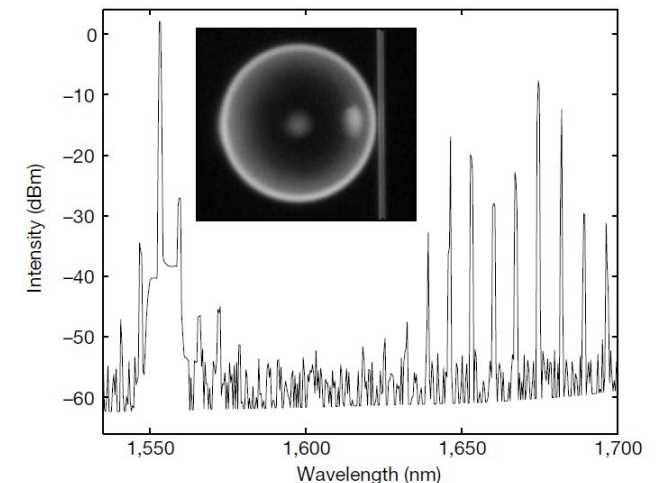
- Third-order nonlinearity  $\chi^3$
- Competing FWM process
- **Silica has broad Raman gain**



D. Hollenbeck, and C. Cantrell, JOSA B **19**, 2886 (2002).

- Raman lasing in high-Q silica microcavity

- small input power ( $\sim \mu\text{W}$ )
- high conversion efficiency
- **Often observing Multi-mode lasing**

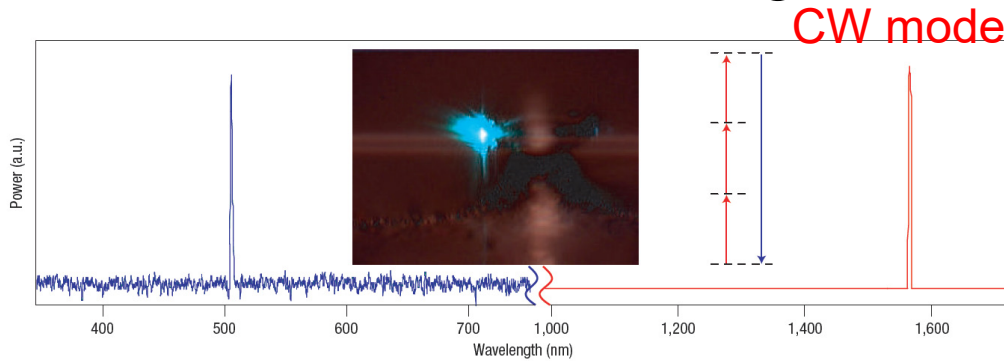


S. Spillane, *et al.*, Nature **415**, 621 (2002).



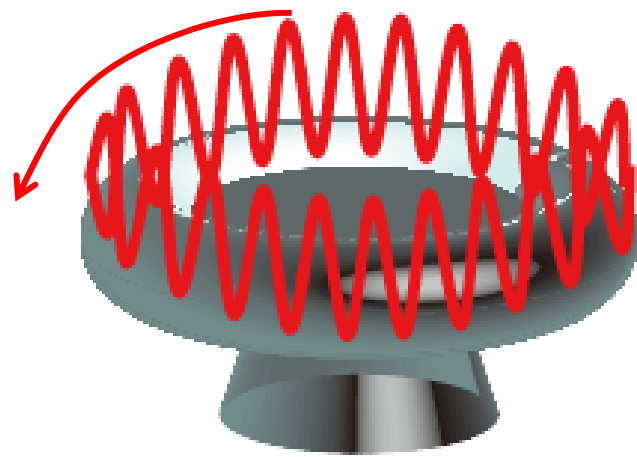
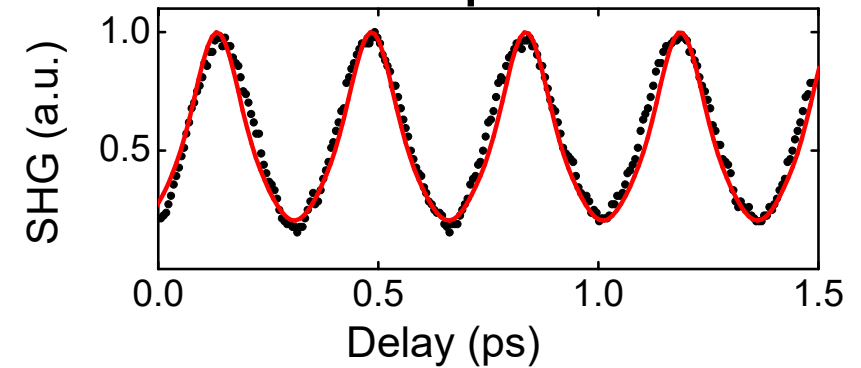
# Visible comb generation with soliton pulse

## ▶ Efficient third-harmonic generation

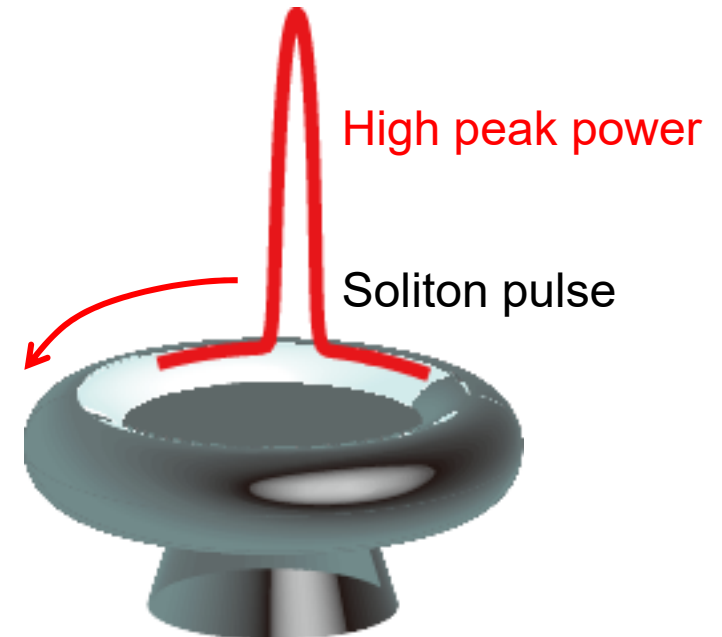


T. Carmon & K. Vahala, Nat. Phys. 3, 430 (2007).

## ▶ THG with pulsed mode



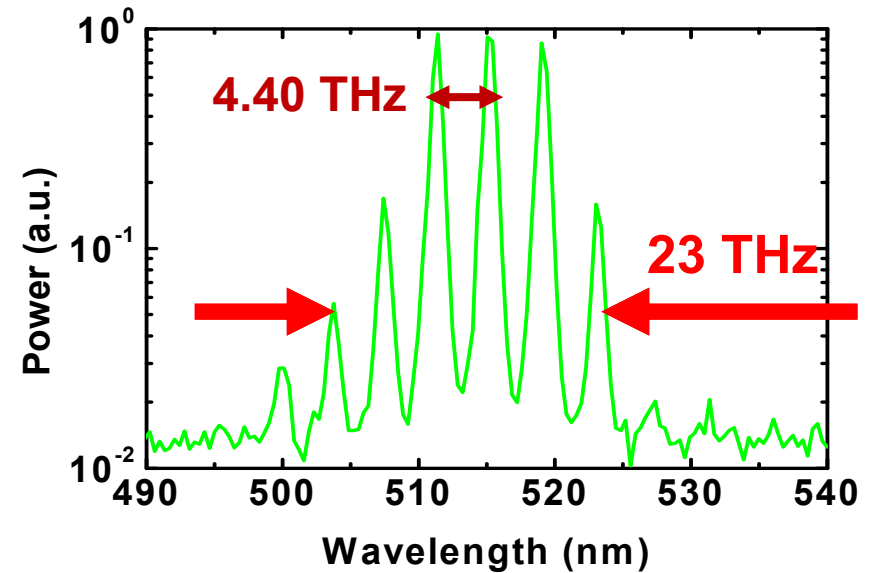
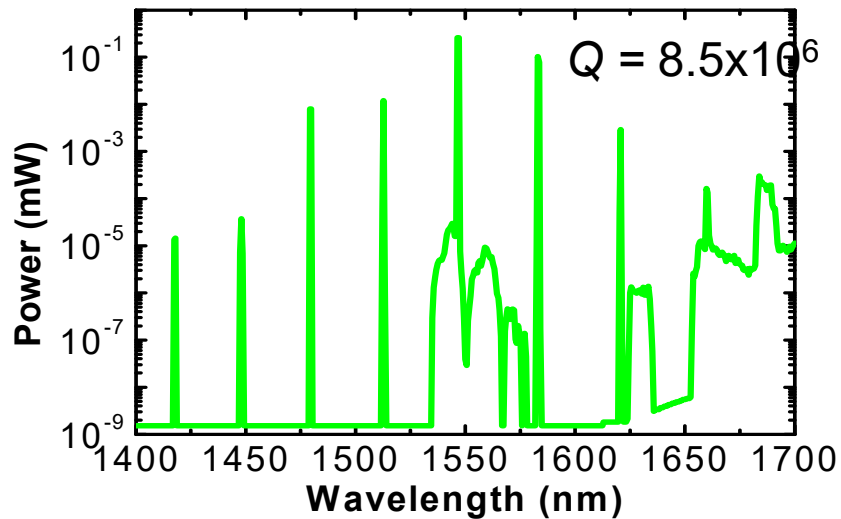
THG generation  
with soliton pulse



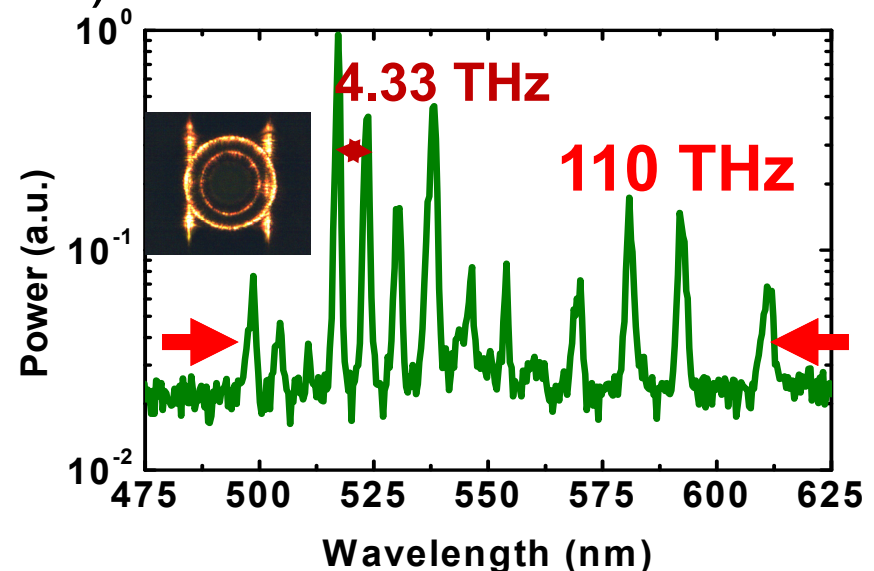
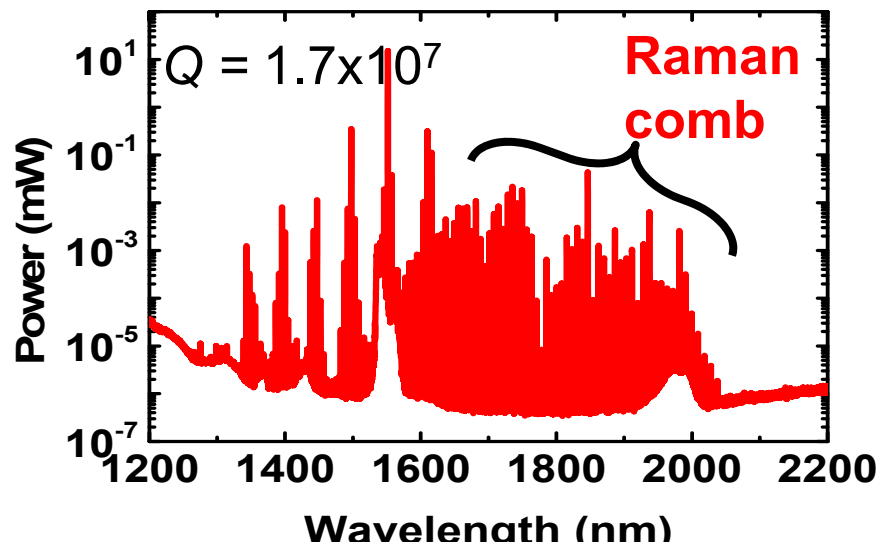
Potential for improving THG efficiency

# Broad bandwidth generation

► w/o Raman comb (Input: 1545.93 nm, 0.94 W)

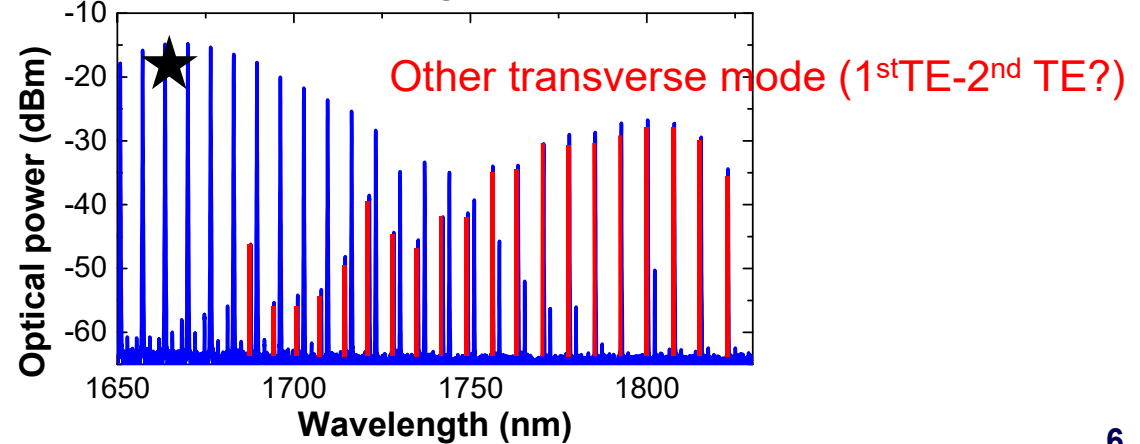
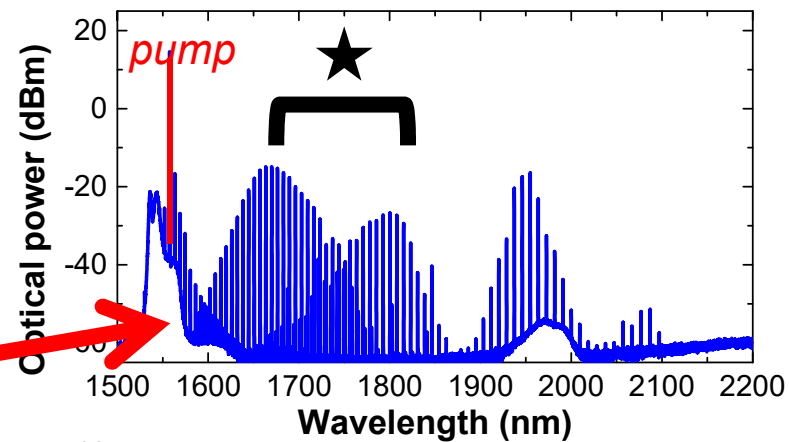
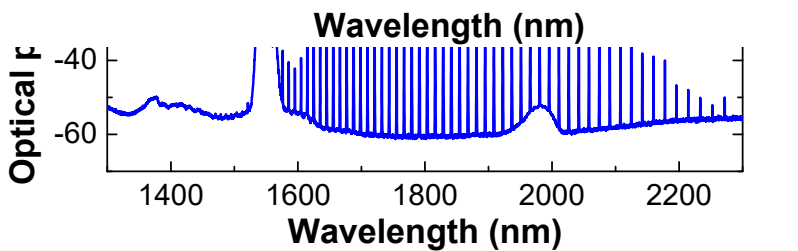
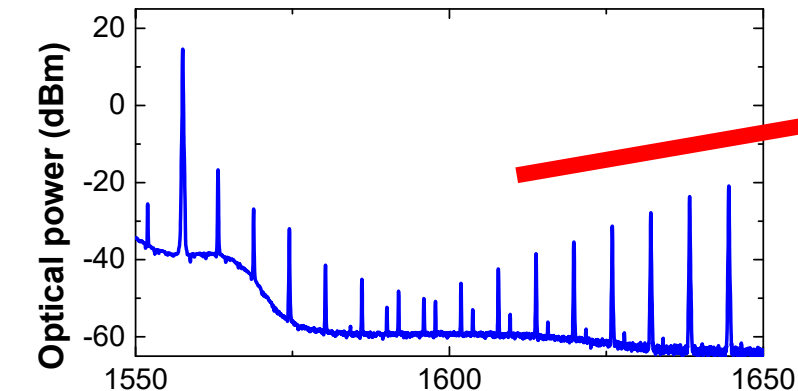
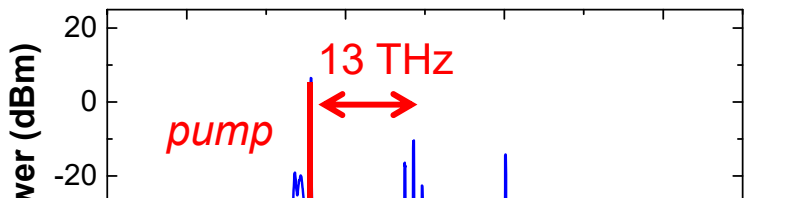
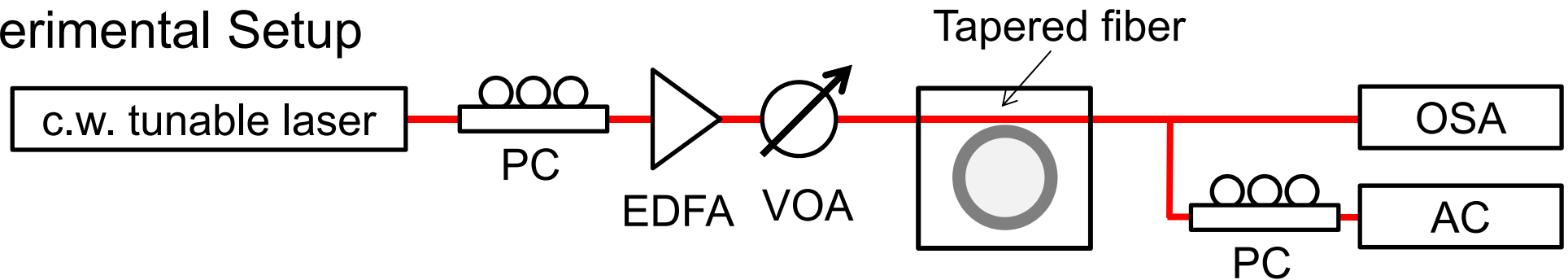


► w/ Raman comb (Input: 1551.59 nm, 1 W)



# Raman scattering in Kerr comb generation

## Experimental Setup



# Motivation & objective

## Motivation:

### Application:

Two-comb modes have possibilities to apply dual comb applications.

### Physics:

Raman scattering in Kerr comb in silica cavity is still unclear.

## Objectives

- ❑ **Develop a numerical model considering 2 modes interacting with Raman scattering.**
- ❑ **Compare numerical/experimental results**



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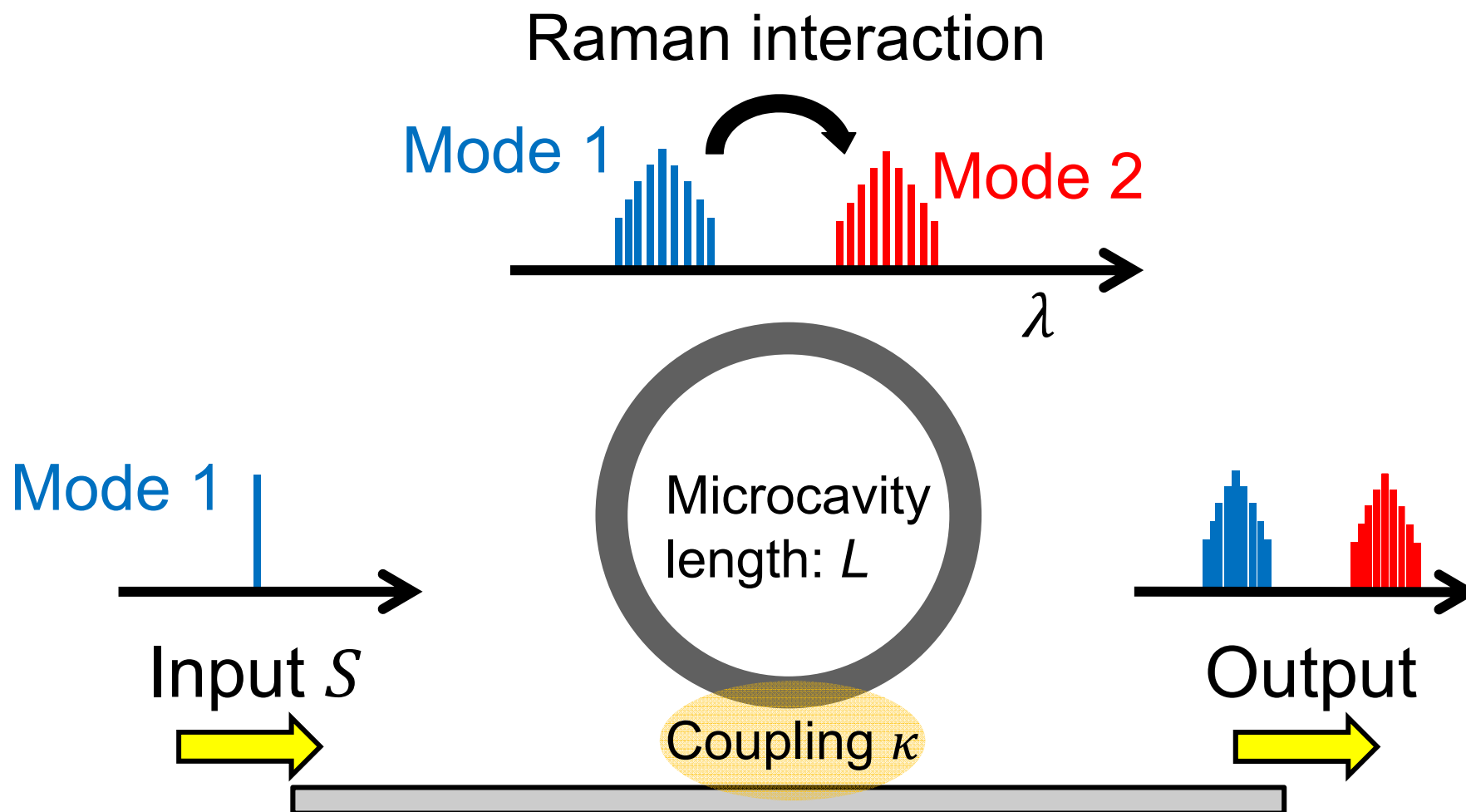
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# Numerical model

- Consider two modes that couple with Raman scattering



# Simultaneous Lugiato-Lefever Equations

□ Lugiato-Lefever equations with Raman interaction

*Mode 1 (pump mode)*

$$t_R \frac{\partial E_p}{\partial t} = \left\{ -\frac{\alpha_p}{2} - \frac{\kappa_p}{2} - i\delta_p + iL \sum_{k \geq 2} \frac{\beta_p^{(k)}}{k!} \left( -i \frac{\partial}{\partial t} \right)^k \right\} E_p + iL \mathbf{N}_p + \sqrt{\kappa_p} S_{\text{in}}$$

$$\mathbf{N}_p = (1 - f_R) \left( \gamma_p |E_p|^2 + 2\Gamma_p |E_s|^2 \right) E_p + f_R \left\{ \begin{array}{l} \gamma_p E_p \int_{-\infty}^{\infty} h_R(t') |E_p(t-t')|^2 dt' \\ + \Gamma_p E_p \int_{-\infty}^{\infty} h_R(t') |E_s(t-t')|^2 dt' + \Gamma_p E_s \int_{-\infty}^{\infty} h_R(t') E_p(t-t') E_s^*(t-t') dt' \end{array} \right\}$$

*Mode 2 (Raman mode)*

$$t_R \frac{\partial E_s}{\partial t} = \left\{ -\frac{\alpha_s}{2} - \frac{\kappa_s}{2} - iL(\beta_s^{(1)} - \beta_p^{(1)}) \left( -i \frac{\partial}{\partial t} \right) + iL \sum_{k \geq 2} \frac{\beta_s^{(k)}}{k!} \left( -i \frac{\partial}{\partial t} \right)^k \right\} E_s + iL \mathbf{N}_s$$

$$\mathbf{N}_s = (1 - f_R) \left( \gamma_s |E_s|^2 + 2\Gamma_s |E_p|^2 \right) E_s + f_R \left\{ \begin{array}{l} \gamma_s E_s \int_{-\infty}^{\infty} h_R(t') |E_s(t-t')|^2 dt' \\ + \Gamma_s E_s \int_{-\infty}^{\infty} h_R(t') |E_p(t-t')|^2 dt' + \Gamma_s E_p \int_{-\infty}^{\infty} h_R(t') E_s(t-t') E_p^*(t-t') dt' \end{array} \right\}$$



# Simultaneous Lugiato-Lefever Equations

□ Lugiato-Lefever equations with Raman interaction

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$$\mathbf{N}_p = (1 - f_R) \left( \gamma_p |E_p|^2 + 2\Gamma_p |E_s|^2 \right) E_p + f_R \left\{ \begin{array}{l} \gamma_p E_p \int_{-\infty}^{\infty} h_R(t') |E_p(t-t')|^2 dt' \\ + \Gamma_p E_p \int_{-\infty}^{\infty} h_R(t') |E_s(t-t')|^2 dt' + \Gamma_p E_s \int_{-\infty}^{\infty} h_R(t') E_p(t-t') E_s^*(t-t') dt' \end{array} \right\}$$

$t_R$ : round trip

$\alpha_p$ : intrinsic loss

$\kappa_p$ : external loss

$\delta_p$ : detuning

$\beta^{(2)}$ : 2<sup>nd</sup> order dispersion

$L$ : cavity length

$N_p$ : nonlinear term

$S_{\text{in}}$ : input power

$f_R$ : contribution of Raman

$\gamma, \Gamma$ : nonlinear coefficient

$h_R$ : Raman response function



# Simultaneous Lugiato-Lefever Equations

## □ Lugiato-Lefever equations with Raman interaction

$t_R$ : round trip

$\alpha_p$ : intrinsic loss

$\kappa_p$ : external loss

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$\beta^{(1)}$ : 1<sup>st</sup> order dispersion

$\beta^{(2)}$ : 2<sup>nd</sup> order dispersion

$L$ : cavity length

$N_p$ : nonlinear term

$S_{in}$ : input power

$f_R$ : contribution of Raman

$\gamma, \Gamma$ : nonlinear coefficient

$h_R$ : Raman response function

## Mode 2 (Raman mode)

$$t_R \frac{\partial E_s}{\partial t} = \left\{ -\frac{\alpha_s}{2} - \frac{\kappa_s}{2} - iL(\beta_s^{(1)} - \beta_p^{(1)}) \left( -i \frac{\partial}{\partial t} \right) + iL \sum_{k \geq 2} \frac{\beta_s^{(k)}}{k!} \left( -i \frac{\partial}{\partial t} \right)^k \right\} E_s + iL N_s$$

$$N_s = (1 - f_R) \left( \gamma_s |E_s|^2 + 2\Gamma_s |E_p|^2 \right) E_s + f_R \left\{ \begin{array}{l} \gamma_s E_s \int_{-\infty}^{\infty} h_R(t') |E_s(t-t')|^2 dt' \\ + \Gamma_s E_s \int_{-\infty}^{\infty} h_R(t') |E_p(t-t')|^2 dt' + \Gamma_s E_p \int_{-\infty}^{\infty} h_R(t') E_s(t-t') E_p^*(t-t') dt' \end{array} \right\}$$



# Supplements: nonlinear coefficient

$$\gamma = \frac{n_2 \omega}{c A_{AA}}$$

$$\Gamma = \frac{n_2 \omega}{c A_{AB}}$$

$n_2$ : nonlinear refractive index

$A_{AA}$ : effective mode area (self)

$A_{AB}$ : effective mode area (interaction)

$$A_{AA} = \frac{\iint |A(x, y)|^2 dx dy * \iint |A(x, y)|^2 dx dy}{\iint |A(x, y)|^4 dx dy}$$

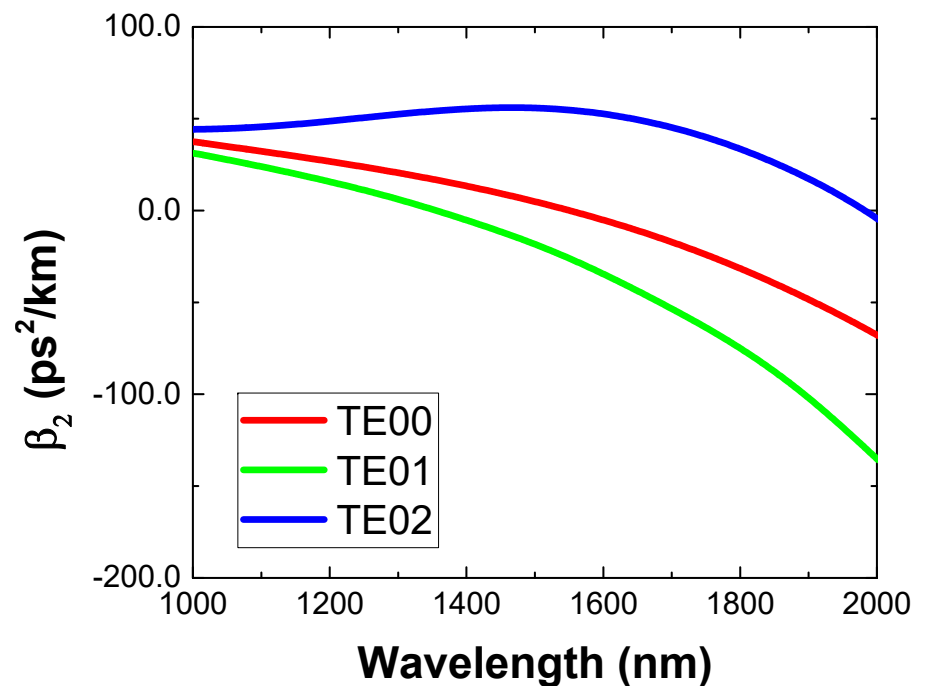
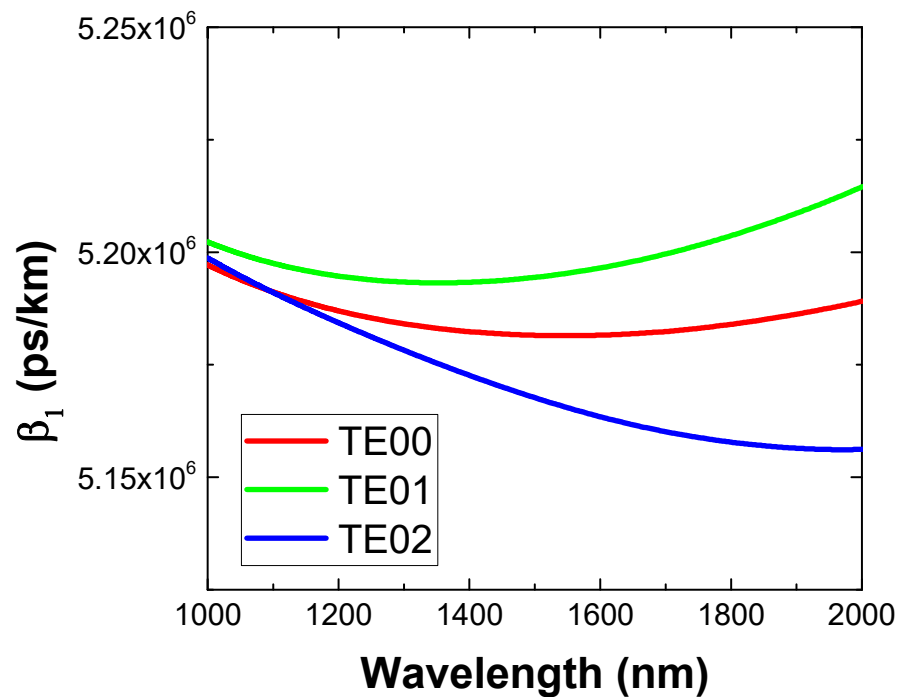
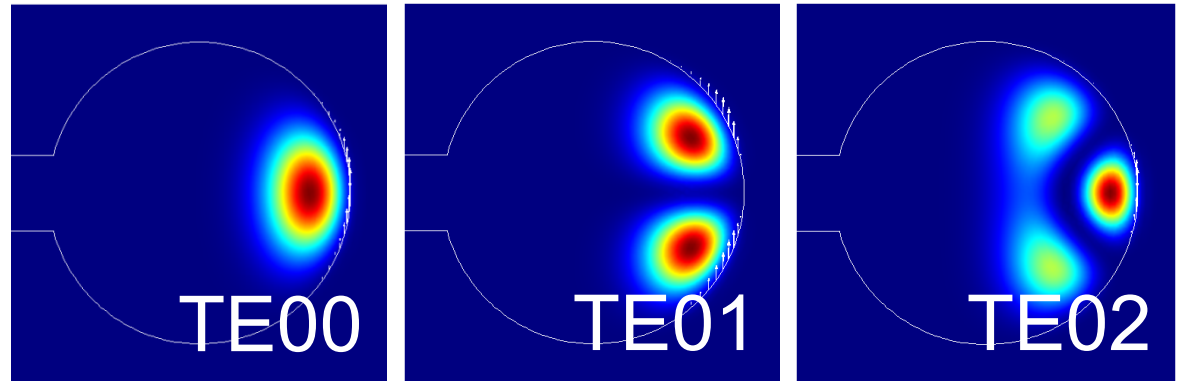
$$A_{AB} = \frac{\iint |A(x, y)|^2 dx dy * \iint |B(x, y)|^2 dx dy}{\iint |A(x, y)|^2 |B(x, y)|^2 dx dy}$$

	Mode area ( $\mu\text{m}^2$ )
$A_{TE00}$	8.285
$A_{TE01}$	10.879
$A_{TE02}$	15.066
$A_{TE00\_TE01}$	15.632
$A_{TE01\_TE02}$	23.532
$A_{TE02\_TE00}$	18.498



# Parameters

Model: Silica toroid  
Major  $R$ : 50  $\mu\text{m}$   
Minor  $R$ : 4  $\mu\text{m}$   
FSR: 600 GHz



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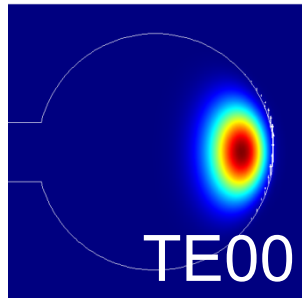
# Dependence on Q factor

$$P_{in} = 100 \text{ mW}$$

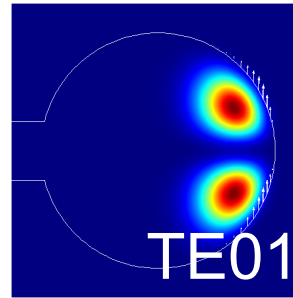
$$\delta_p = 1 \times 10^{-4}$$

Pump mode:  $TE_{00}$

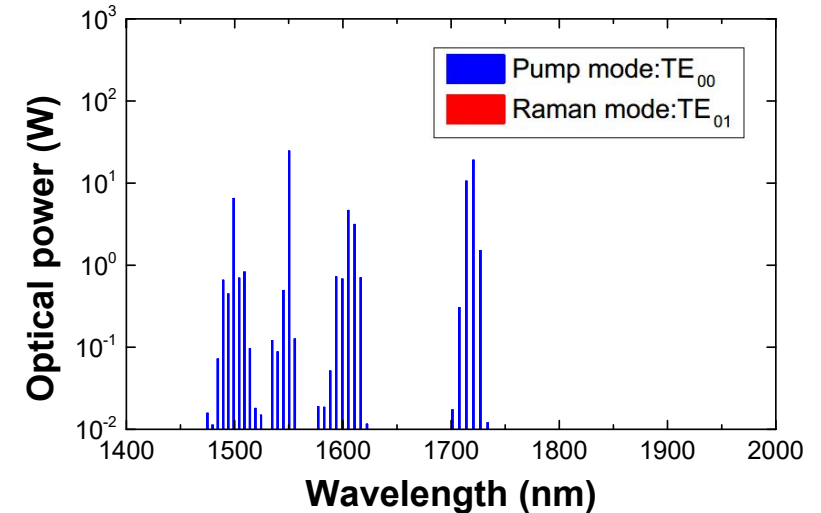
Raman mode:  $TE_{01}$



$$Q_{TE_{00}} = 1 \times 10^8$$

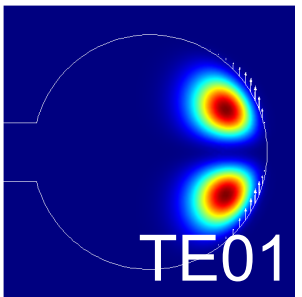


$$Q_{TE_{01}} = 2 \times 10^7$$

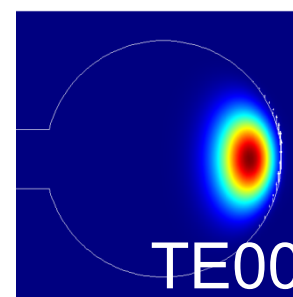


Pump mode:  $TE_{01}$

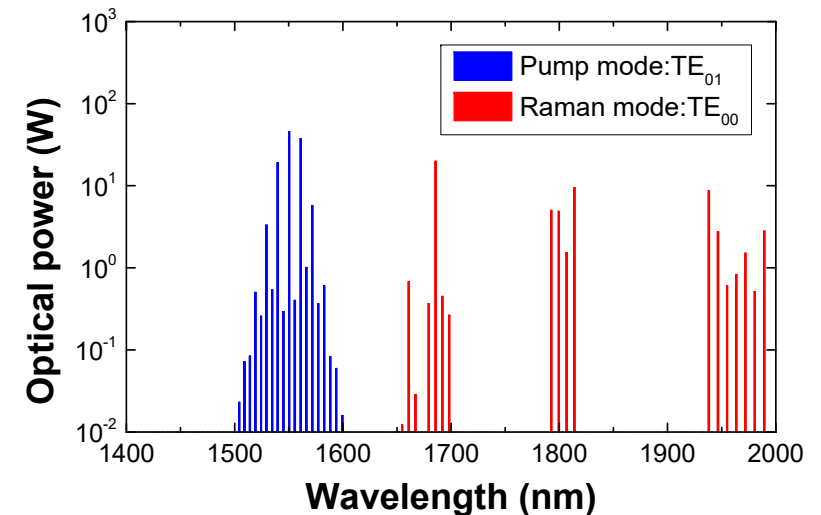
Raman mode:  $TE_{00}$



$$Q_{TE_{01}} = 2 \times 10^7$$



$$Q_{TE_{00}} = 1 \times 10^8$$



★ Lower Q mode excites Higher Q mode Raman scattering.

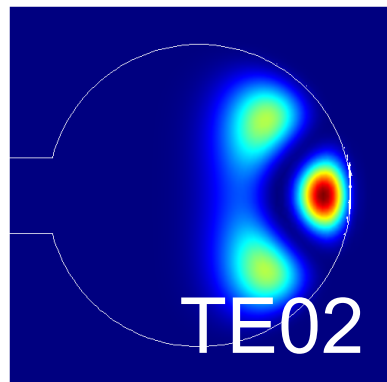


# Comparison with the experimental result

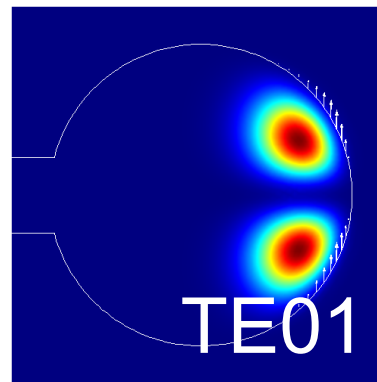
Pump mode:  $TE_{02}$

Raman mode:  $TE_{01}$

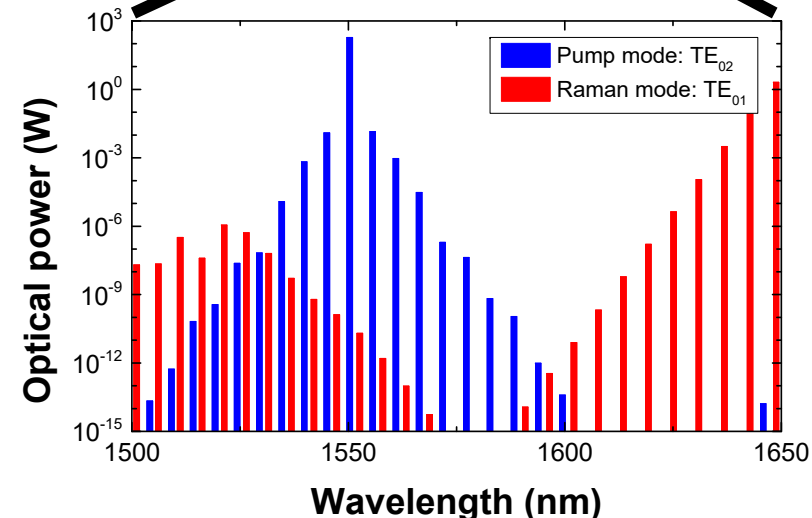
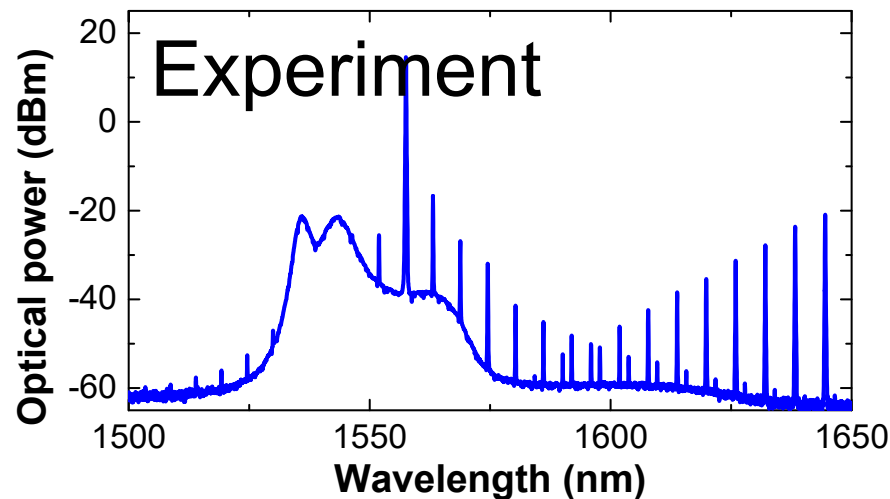
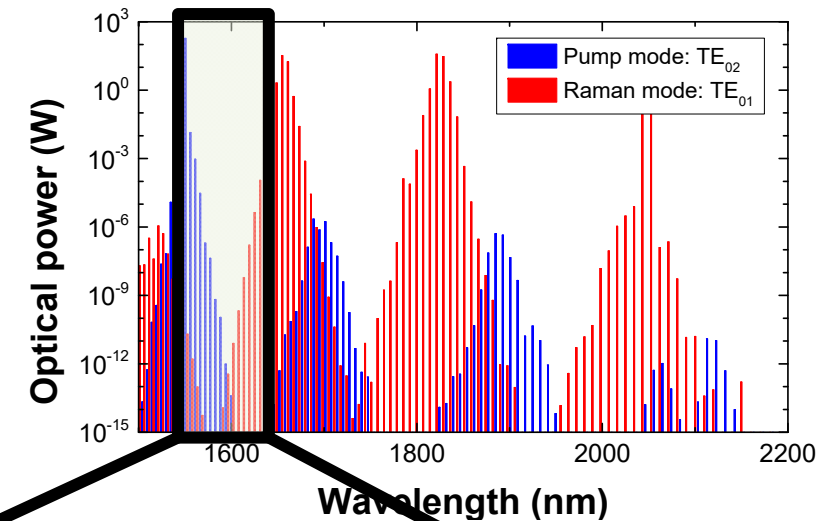
$$P_{in} = 1 \text{ W}, \delta_p = 3.575 \times 10^{-4}$$



$$Q_{TE_{02}} = 7 \times 10^6$$

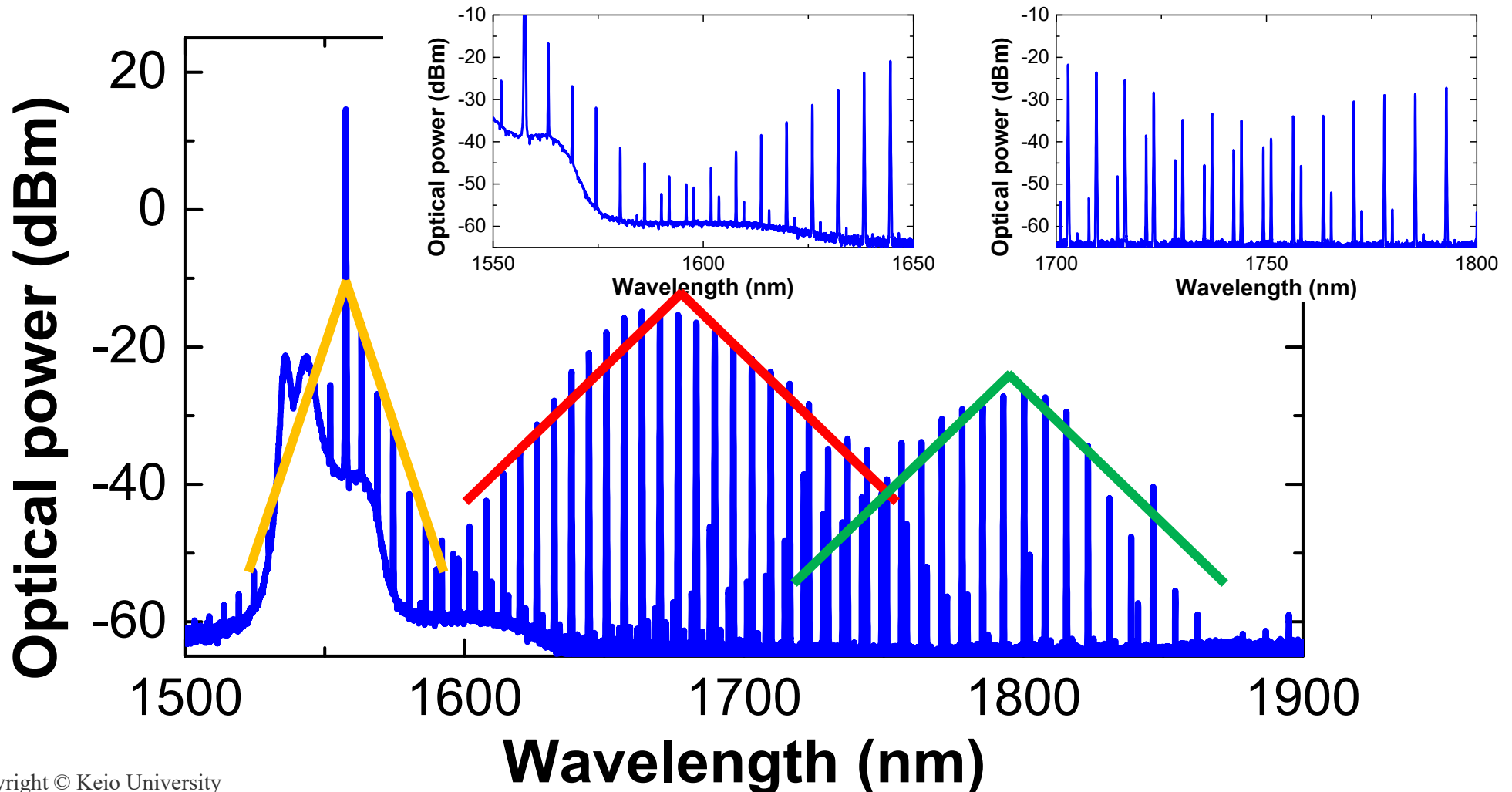


$$Q_{TE_{01}} = 5 \times 10^7$$



# Three mode interaction

- Three mode interaction should be considered in this system.  
(ex: TE<sub>02</sub>→TE<sub>01</sub>→TE<sub>00</sub>)



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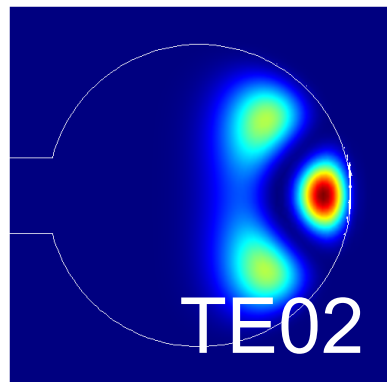


# Raman soliton generation

Pump mode: TE<sub>02</sub>

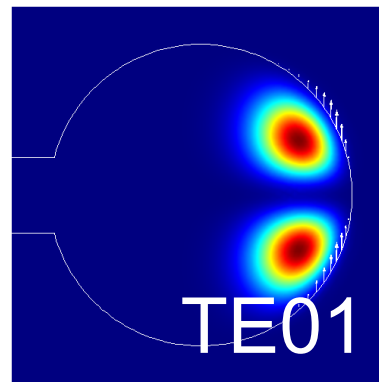
Raman mode: TE<sub>01</sub>

$$P_{\text{in}} = 1 \text{ W}, \delta_p = 3.575 \times 10^{-4}$$



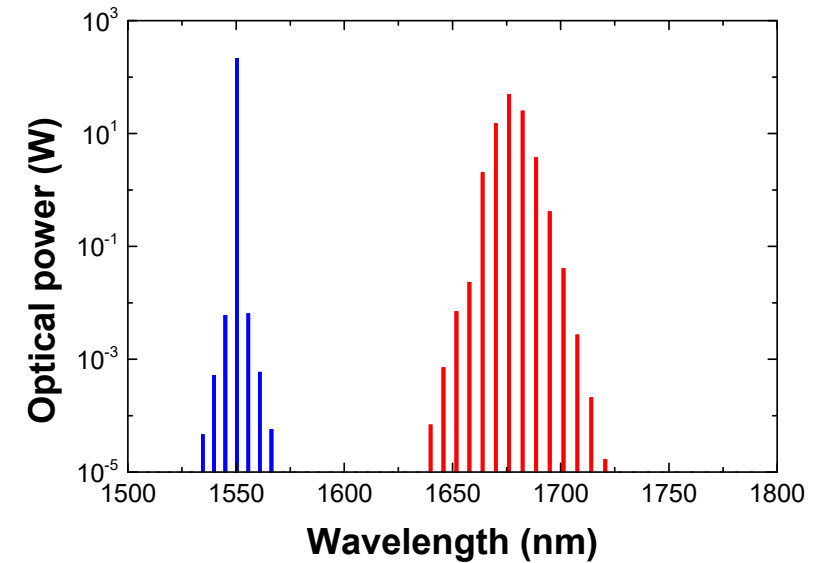
TE02

$$Q_{\text{TE}_{02}} = 7 \times 10^6$$

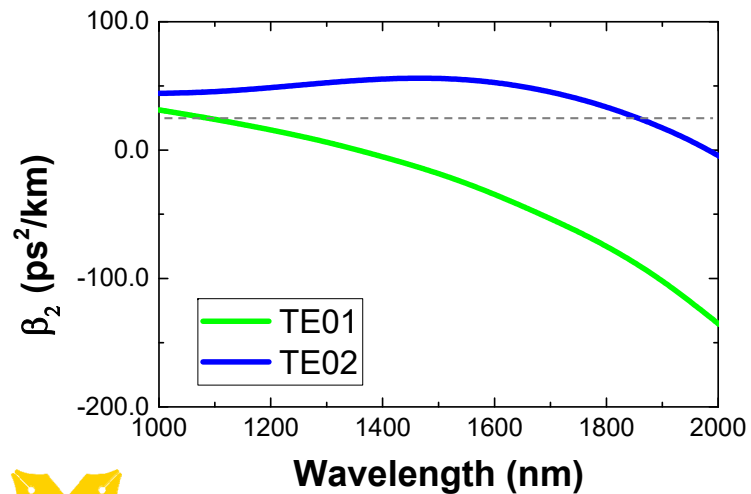


TE01

$$Q_{\text{TE}_{01}} = 1.8 \times 10^7$$

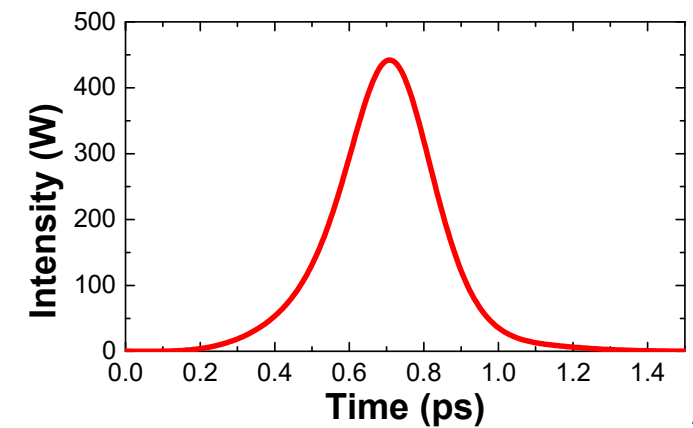
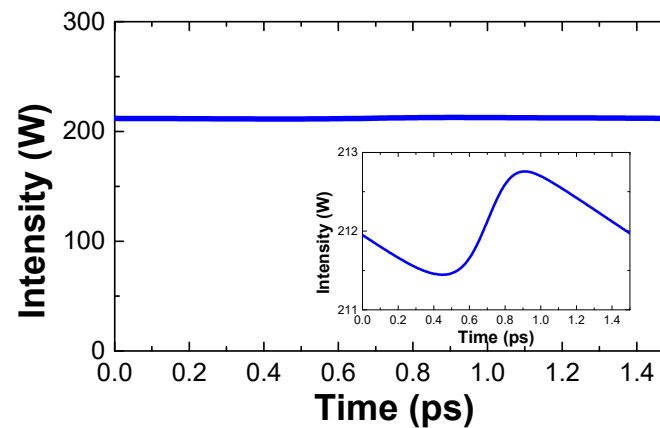


Wavelength (nm)



Pump mode

Raman mode



# Raman soliton generation

## □ Chaotic condition (too high-Q)

Pump mode:  $TE_{02}$       Raman mode:  $TE_{01}$

$$Q_{TE_{02}} = 7 \times 10^6$$

$$Q_{TE_{01}} = 5.0 \times 10^8$$

## □ Soliton condition (reasonable Q)

Pump mode:  $TE_{02}$       Raman mode:  $TE_{01}$

$$Q_{TE_{02}} = 7 \times 10^6$$

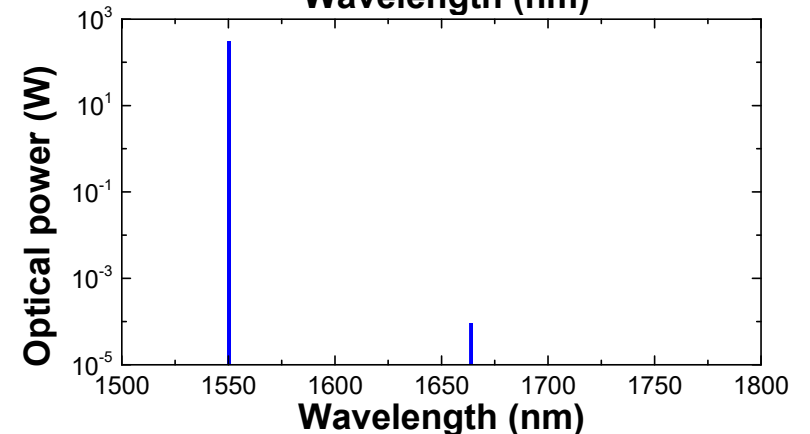
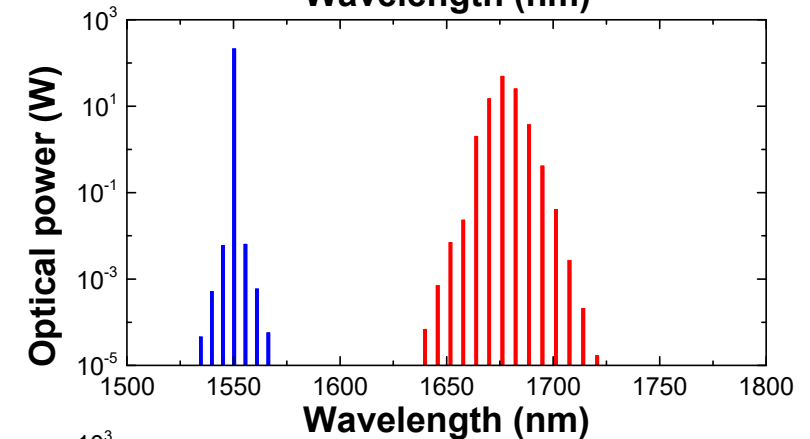
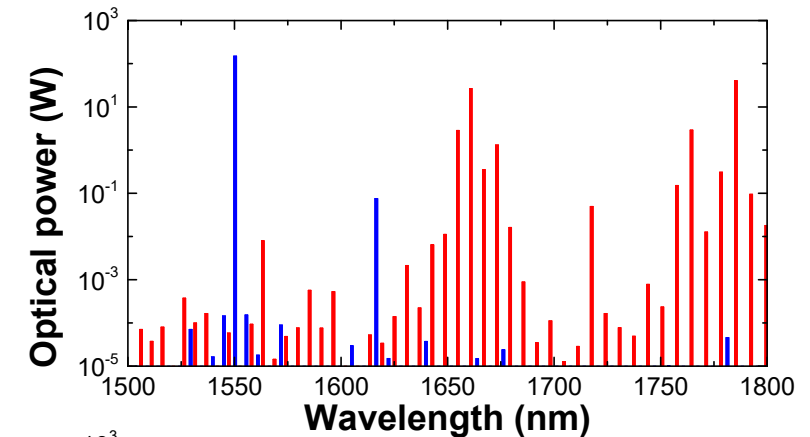
$$Q_{TE_{01}} = 1.8 \times 10^7$$

## □ No excitation (too low-Q)

Pump mode:  $TE_{02}$       Raman mode:  $TE_{01}$

$$Q_{TE_{02}} = 7 \times 10^6$$

$$Q_{TE_{01}} = 9.0 \times 10^6$$



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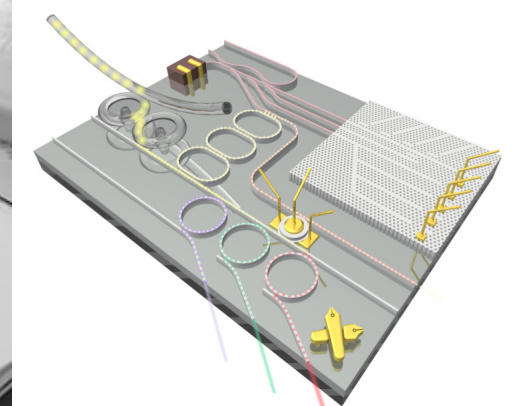
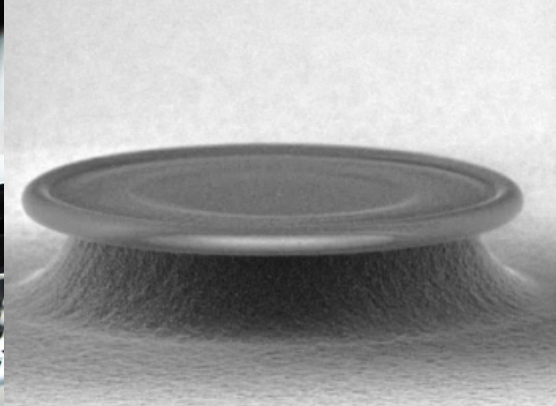
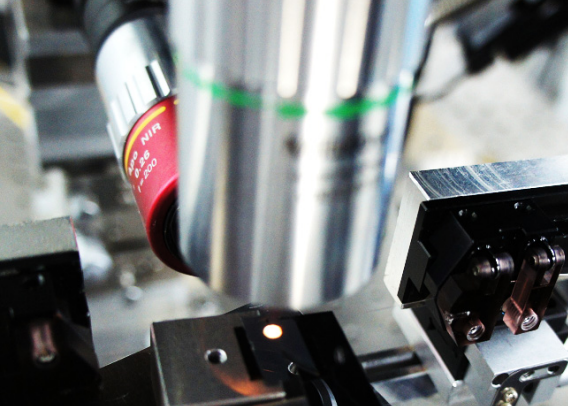
## [Experiment]

- We observed multi-mode generation through Raman scattering with silica toroidal microcavity.

## [Numerical simulation]

- We developed simultaneous LLE considering Raman effect.
- We considered  $TE_{00}$ ,  $TE_{01}$ , and  $TE_{02}$  modes.
  - Lower Q mode pumping excites Higher Q mode.
  - Three mode interaction could be occurred.
  - Raman soliton can achieve when Raman mode satisfies soliton condition.





# Thank you very much

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## Acknowledgement

- Ministry of Education, Culture, Sports, Science and Technology (MEXT) (KAKEN 15H05429)
- Grant-in-Aid from the Ministry of Education, Culture, Sports, Science and Technology, Japan for the Photon Frontier Network Program.