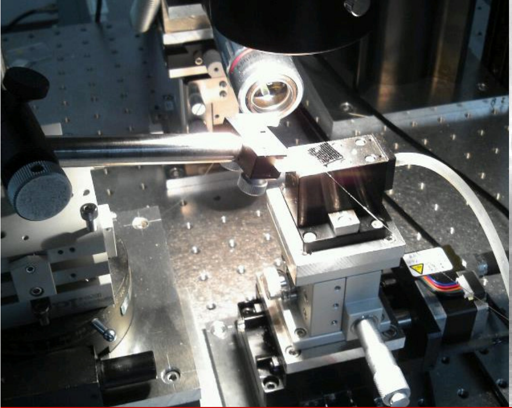
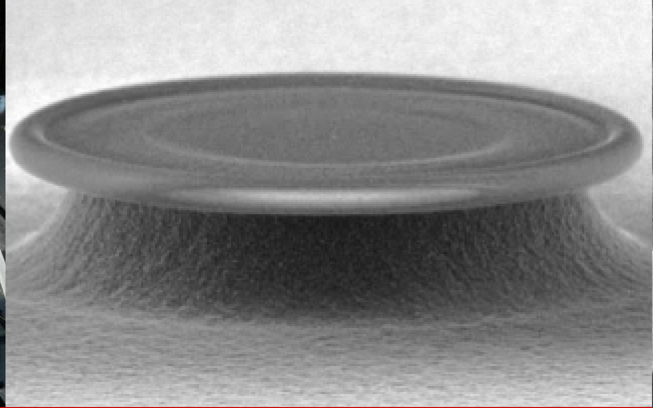
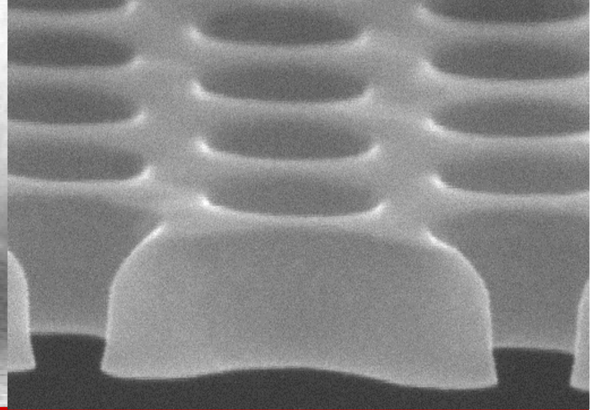
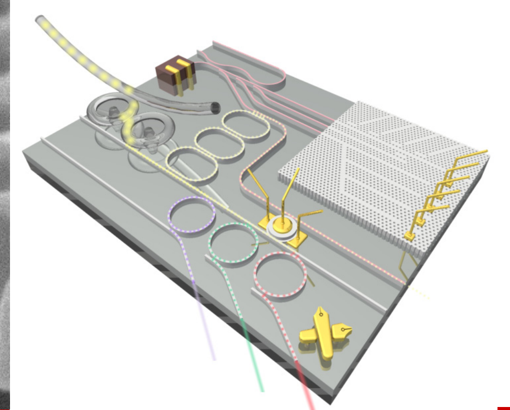

PIERS2016
2P_13-1 13:00-13:20 Aug. 9. 2016

Kerr Comb Generation in a Whispering Gallery Mode Microcavity: The Effect of Mode Coupling

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Outline

1. Background & Motivation
2. Modelling & effect of mode crossing
3. Effect of CW/CCW mode coupling
4. Effect of TM/TE mode coupling
5. Longitudinal mode family coupling via Raman nonlinearity

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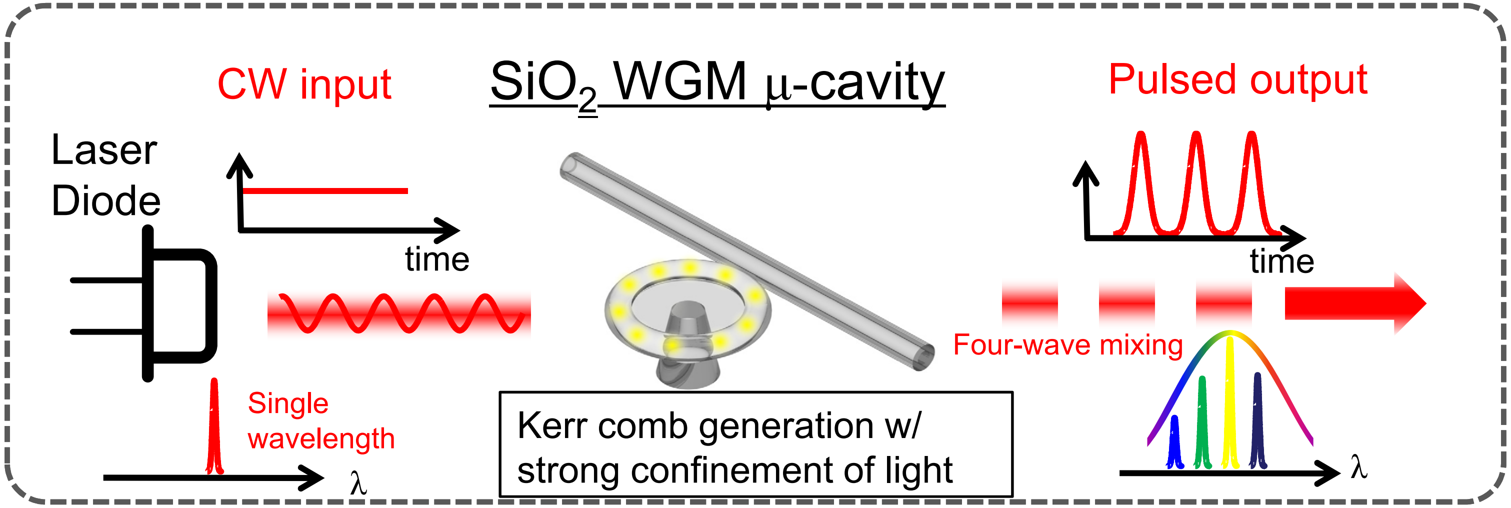
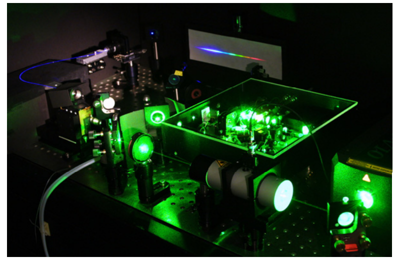
Kerr comb in microcavity system

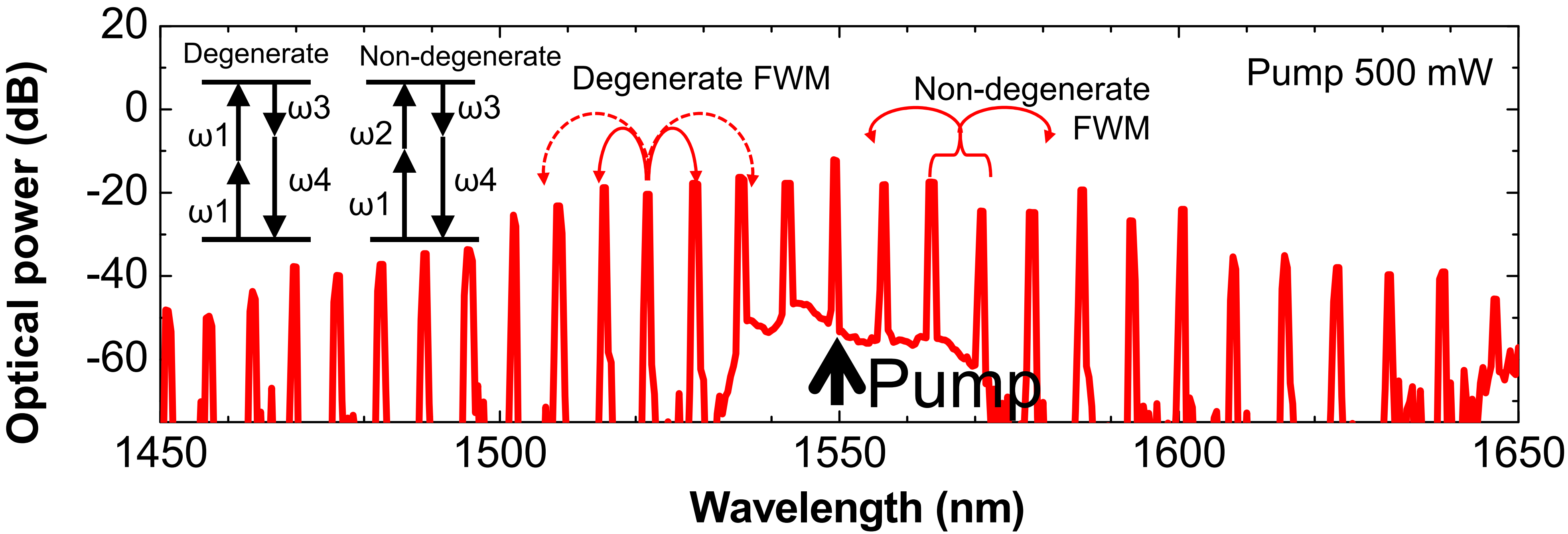
Convert CW laser to ultrashort pulse train w/ >600 GHz repetition rate



Ti:Sapphire laser based comb

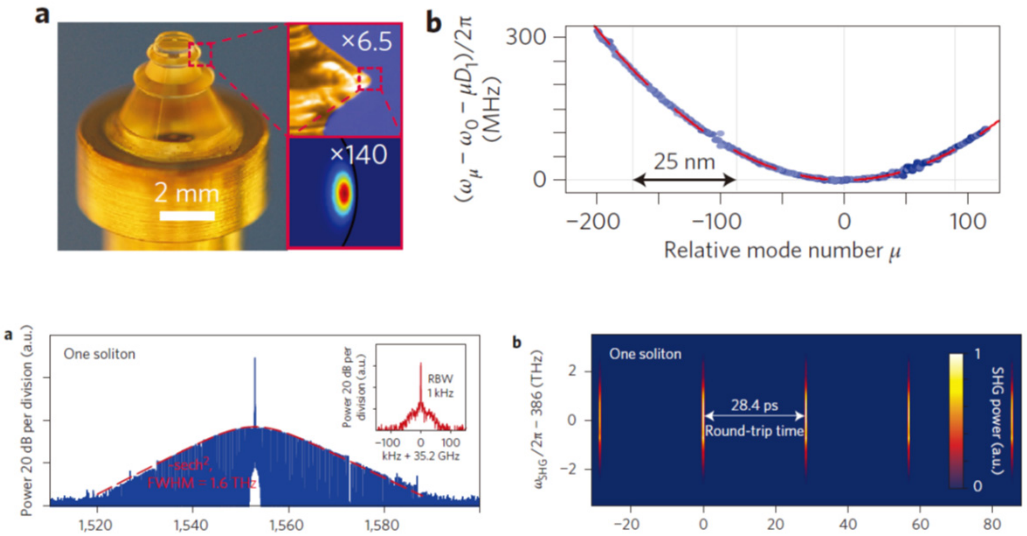


large & expensive



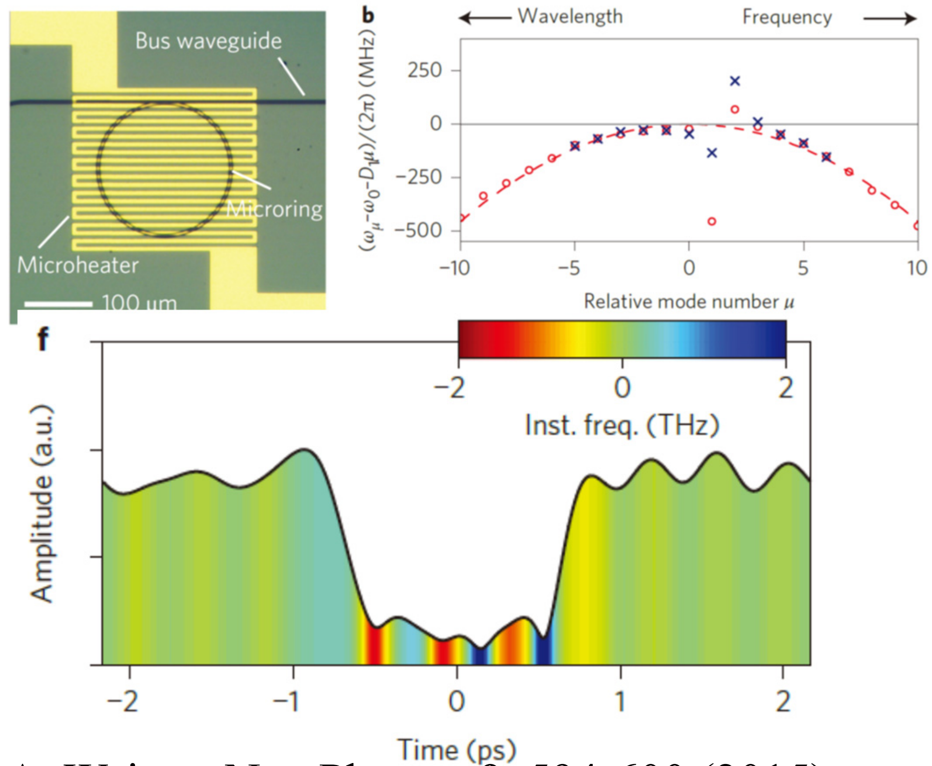
Dispersion and coupled cavity for soliton

▶Soliton formation w/ negative dispersion



T. Herr, *et al*. Nat. Photonics **8**, 145 (2014).

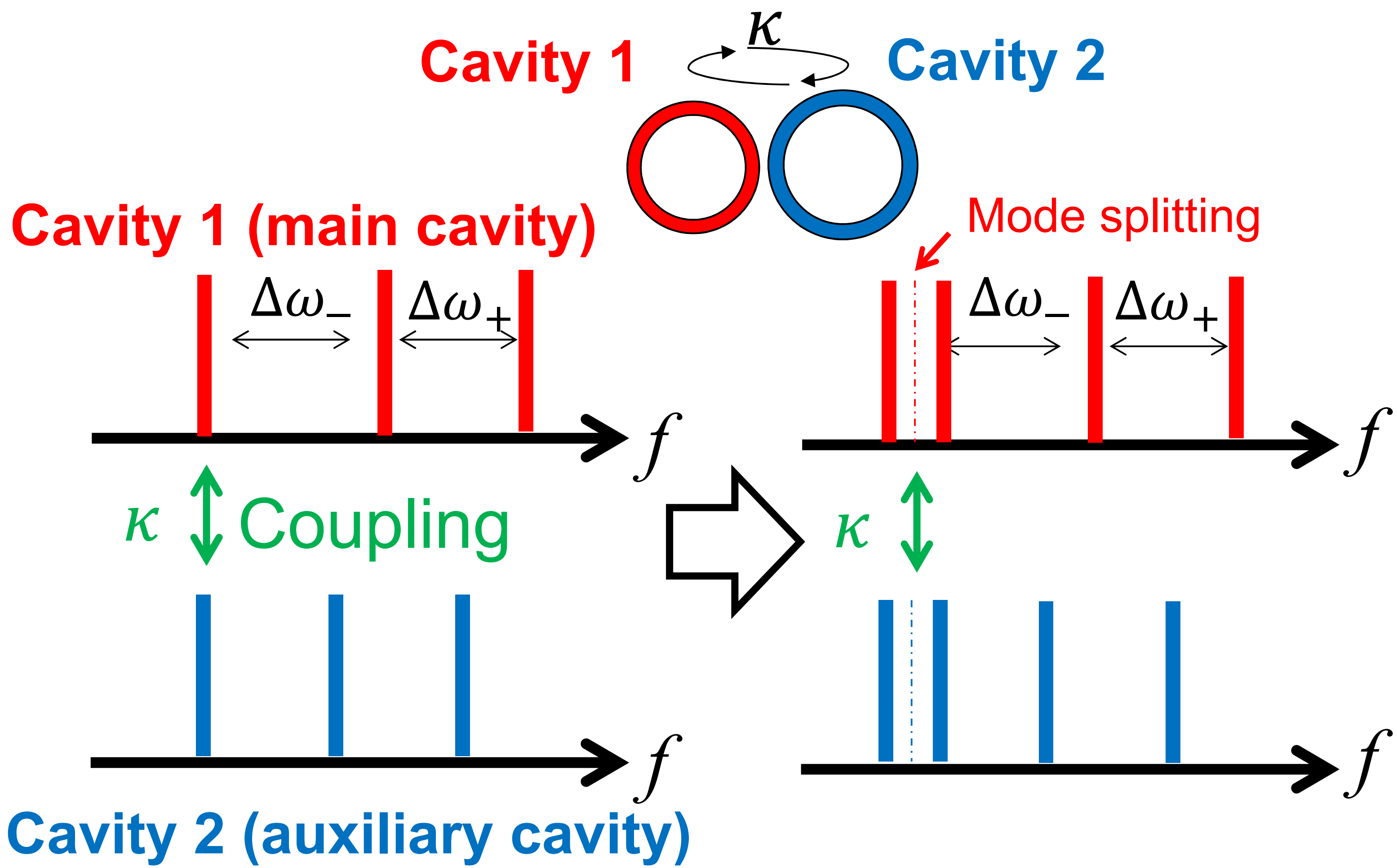
▶Coupled cavity for soliton generation (dark soliton)



A. Weiner: Nat. Photon. **9**, 594-600 (2015)

⇨ Coupled cavity system is a new platform for the generation of Kerr comb

Dispersion control by using mode splitting



Anomalous : $\Delta\omega_+ - \Delta\omega_- > 0$

Normal : $\Delta\omega_+ - \Delta\omega_- < 0$

$$\left(\beta_2 = -\frac{n}{c} \cdot \frac{\Delta\omega_+ - \Delta\omega_-}{4\pi^2 \cdot \mathrm{FSR}^2}\right)$$

⇨ FSR modulated by coupling modes (local dispersion control)

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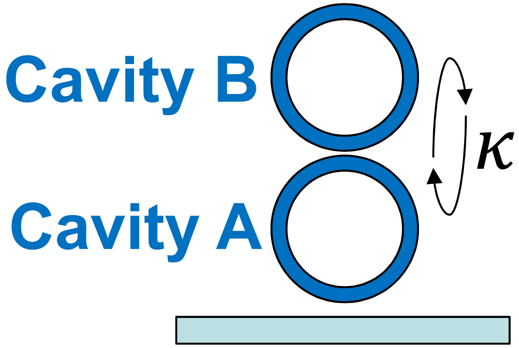
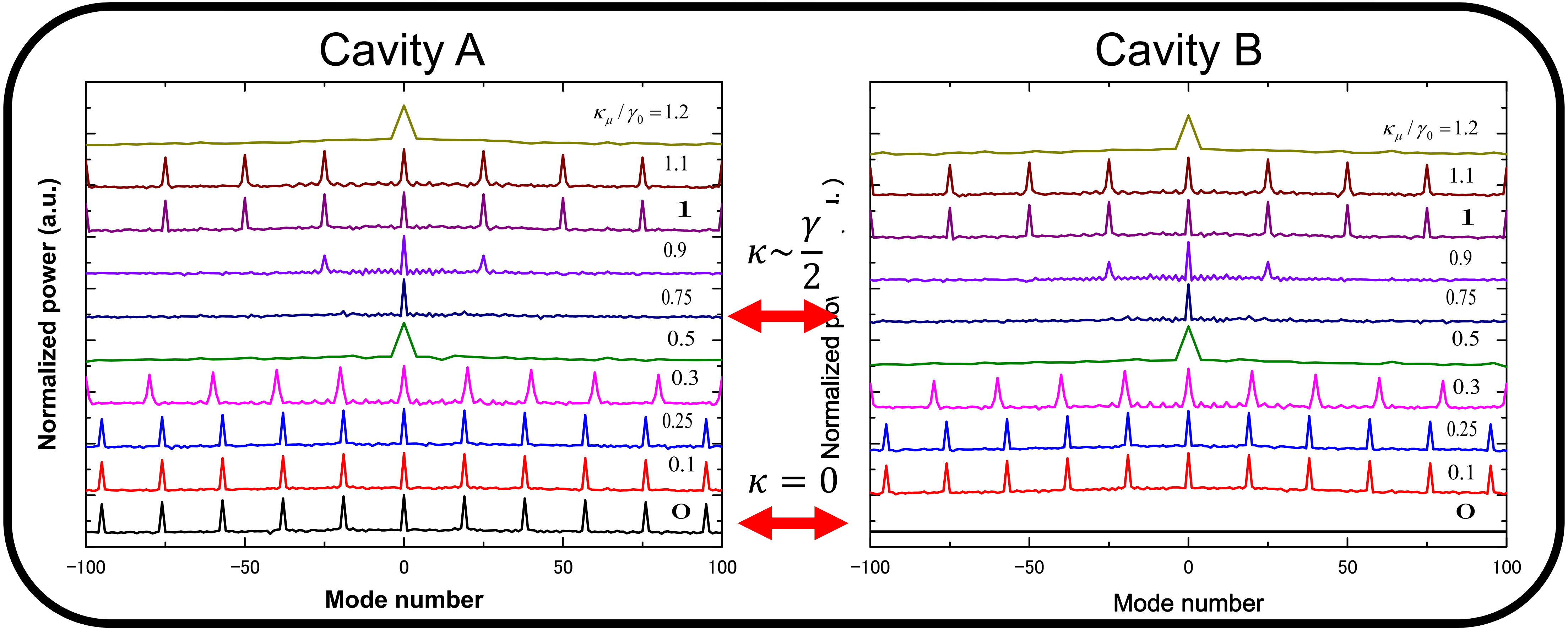
Simple Kerr comb simulation



Loss | FWM | Coupling | Input

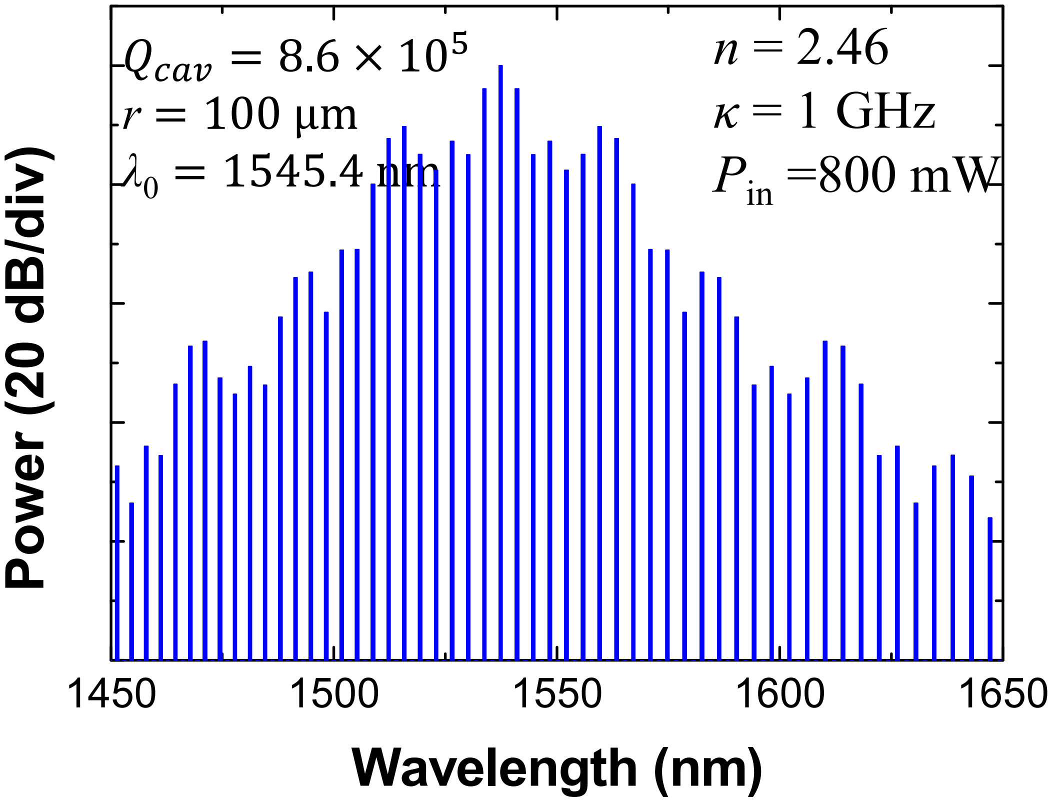
$$\frac{\partial \tilde{A}_\mu}{\partial t} = \boxed{\frac{\gamma_{A_0}}{2}\left(-1 + i\Omega_{A_\mu}\right)\tilde{A}_\mu} + \boxed{\frac{\gamma_{A_0}}{2}\cdot i\sum_{\alpha,\beta,\gamma}\delta_{\mu-(\alpha-\beta+\gamma)}\tilde{A}_\alpha\tilde{A}^*_\beta\tilde{A}_\gamma} + \boxed{i\delta_{\omega_{A_\mu}-\omega_{B_\theta}}\frac{\kappa_\theta}{2}\sqrt{\frac{g_A\gamma_{B_0}}{g_B\gamma_{A_0}}}\tilde{B}_\theta} + \boxed{\frac{\gamma_{A_0}}{2}\delta_\mu f_0}$$

$$\frac{\partial \tilde{B}_\mu}{\partial t} = \boxed{\frac{\gamma_{B_0}}{2}\left(-1 + i\Omega_{B_\mu}\right)\tilde{B}_\mu} + \boxed{\frac{\gamma_{B_0}}{2}\cdot i\sum_{\alpha,\beta,\gamma}\delta_{\mu-(\alpha-\beta+\gamma)}\tilde{B}_\alpha\tilde{B}^*_\beta\tilde{B}_\gamma} + \boxed{\delta_{\omega_{B_\mu}-\omega_{A_\theta}}\frac{\kappa_\theta}{2}\sqrt{\frac{g_B\gamma_{A_0}}{g_A\gamma_{B_0}}}\tilde{A}_\theta}$$

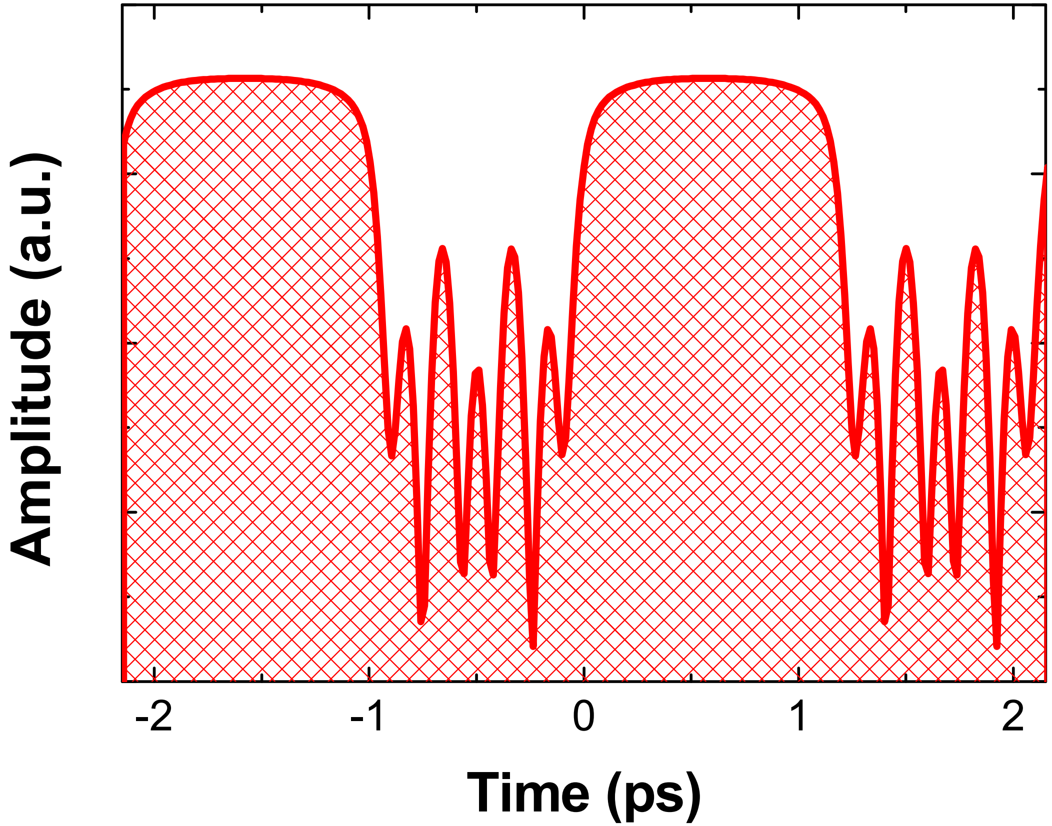


Dark soliton in normal dispersion cavity

▶Spectrum



▶Waveform



Mode crossing is needed to initialize soliton formation

⇨ Dark soliton obtained w/ normal dispersion

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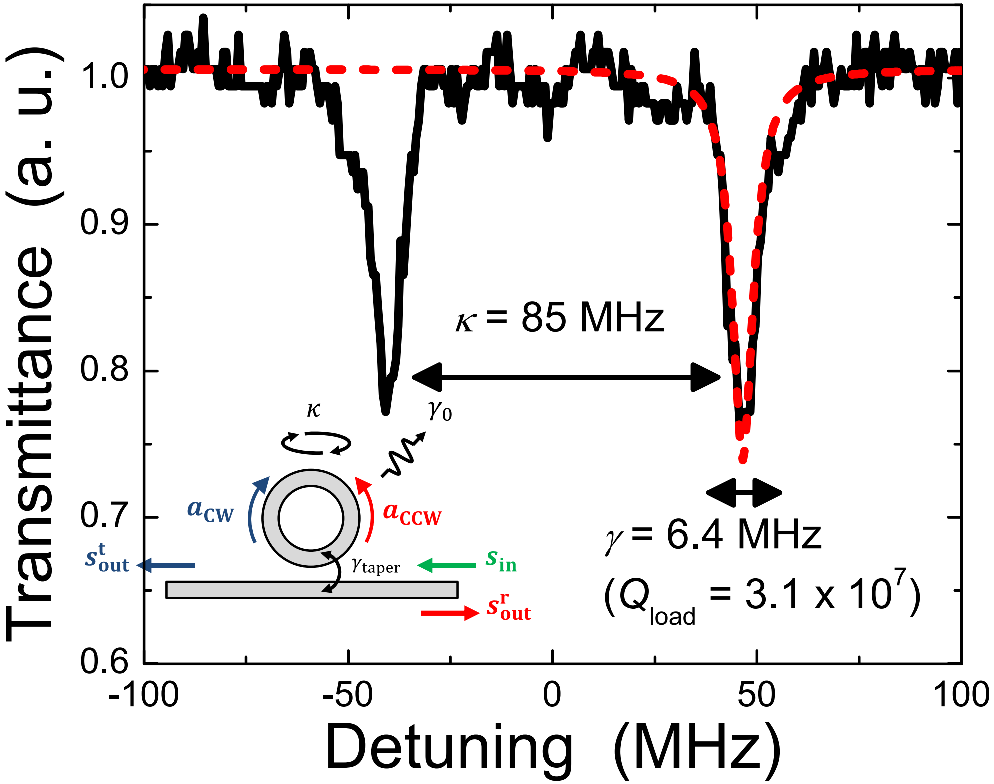
Strong coupling between CW/CCW with small cavity

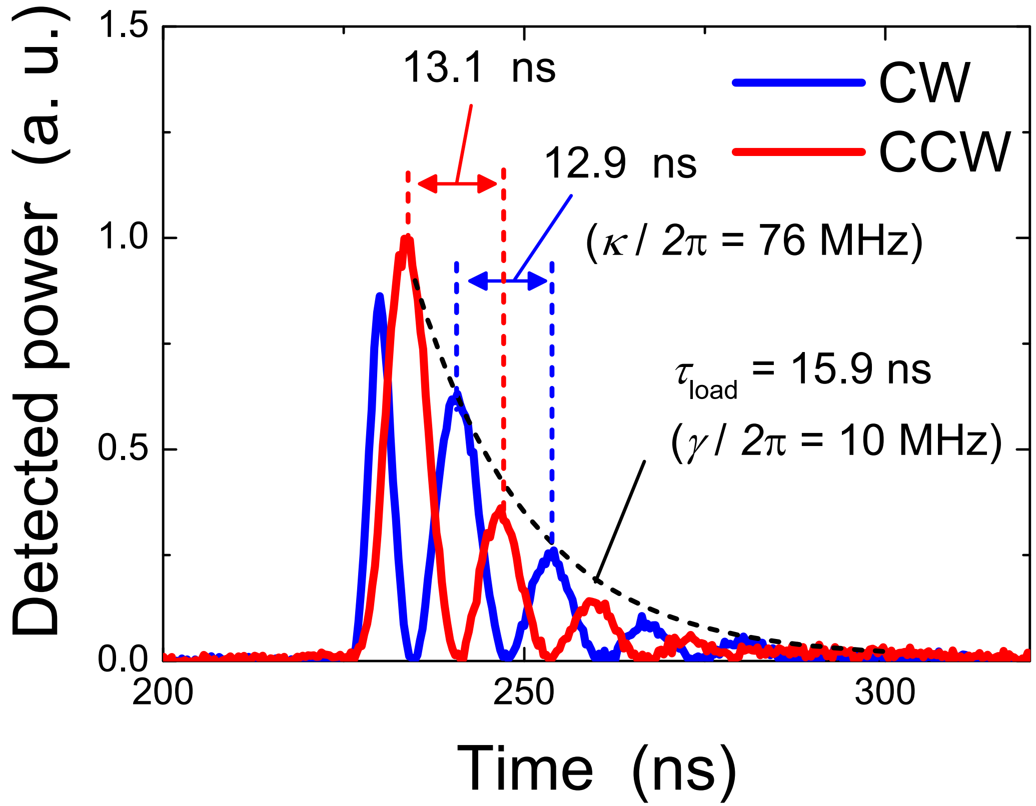
W. Yoshiki, *et al*. Opt. Express, **23**, 30851 (2015).

▶Resonance splitting (Freq. domain)



- Coupling rate: κ
- Decay rate: γ = (γ_0 + γ_{taper})

▶Energy oscillation (Time domain)

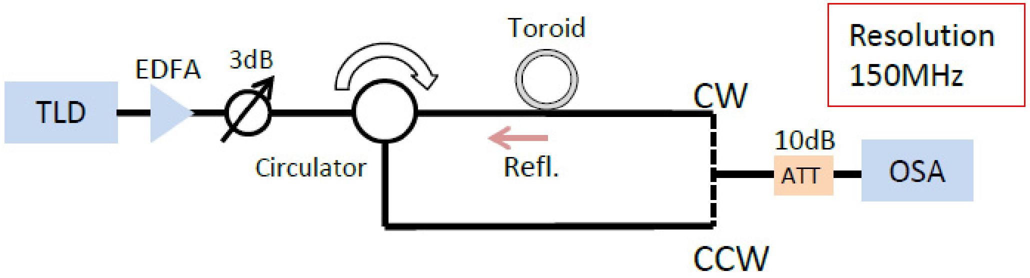


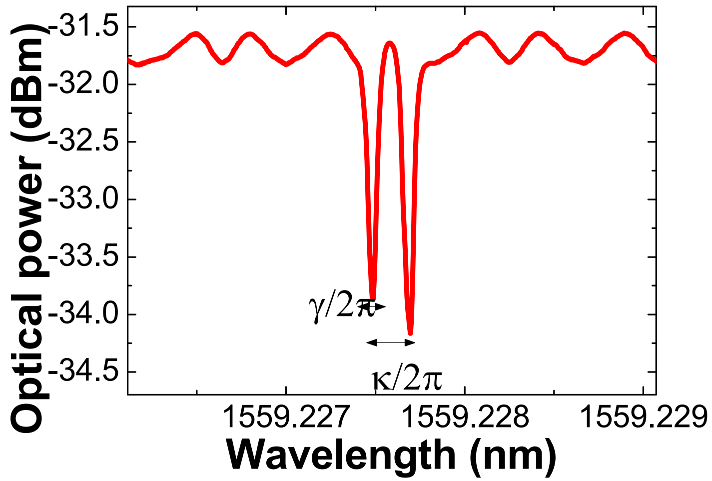
$\kappa/2\pi$ = **85 MHz**, $\gamma/2\pi$ = **6.4 MHz**

-> **$\Gamma \approx 13$**

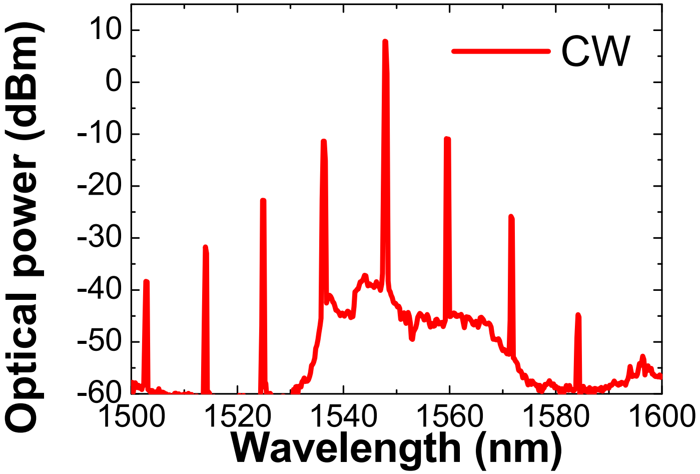
Effect of CW/CCW coupling to Kerr comb

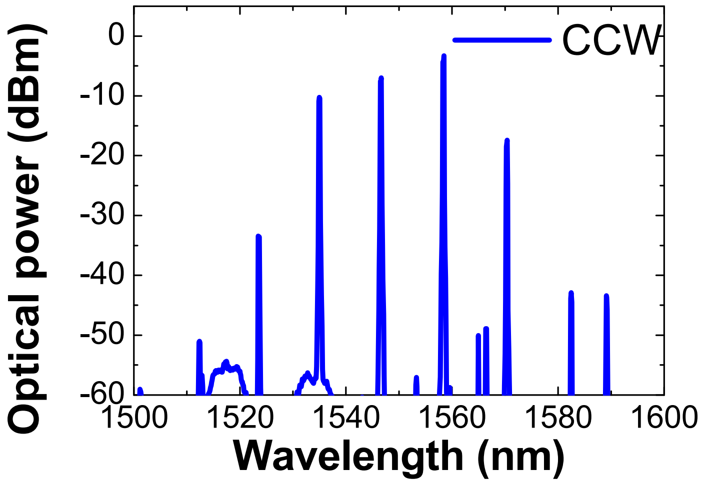
►Experimental setup

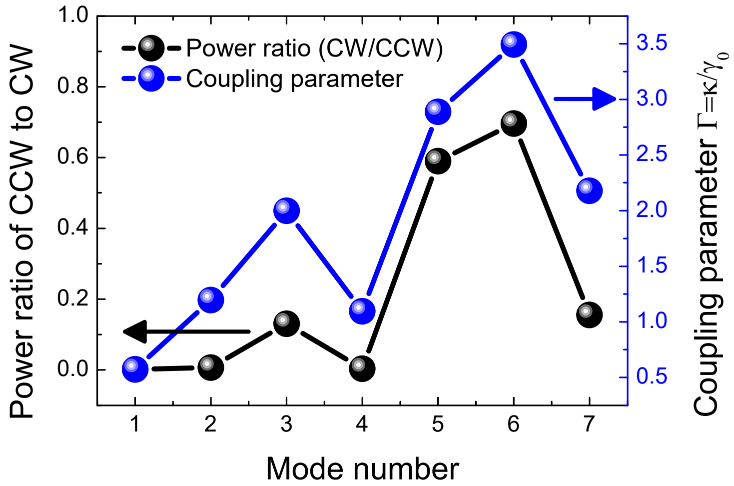




►CW/CCW comb & coupling strength

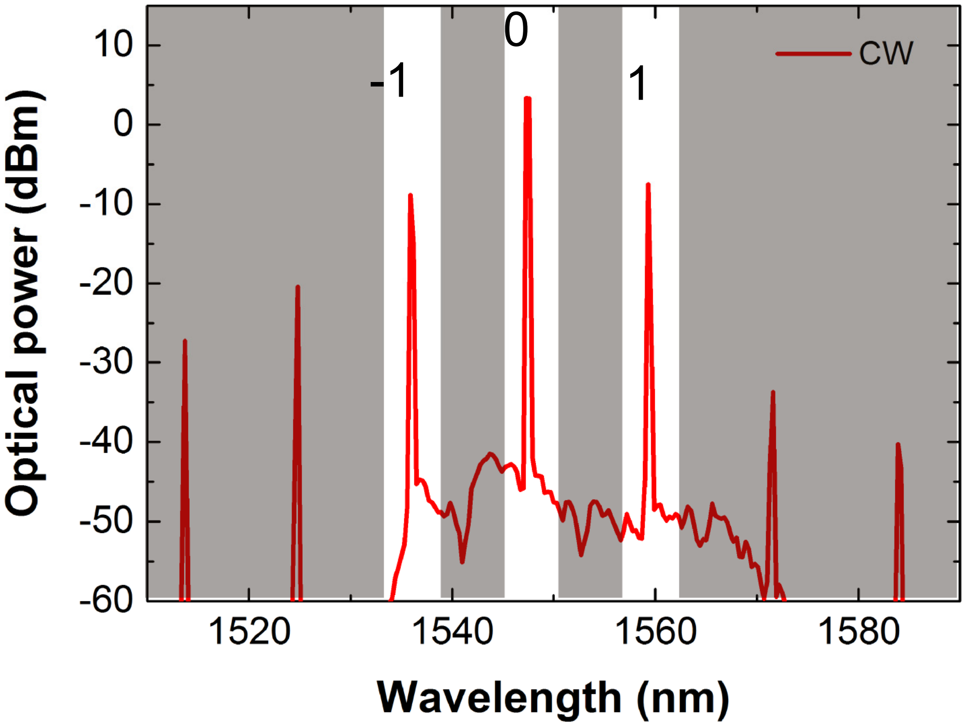






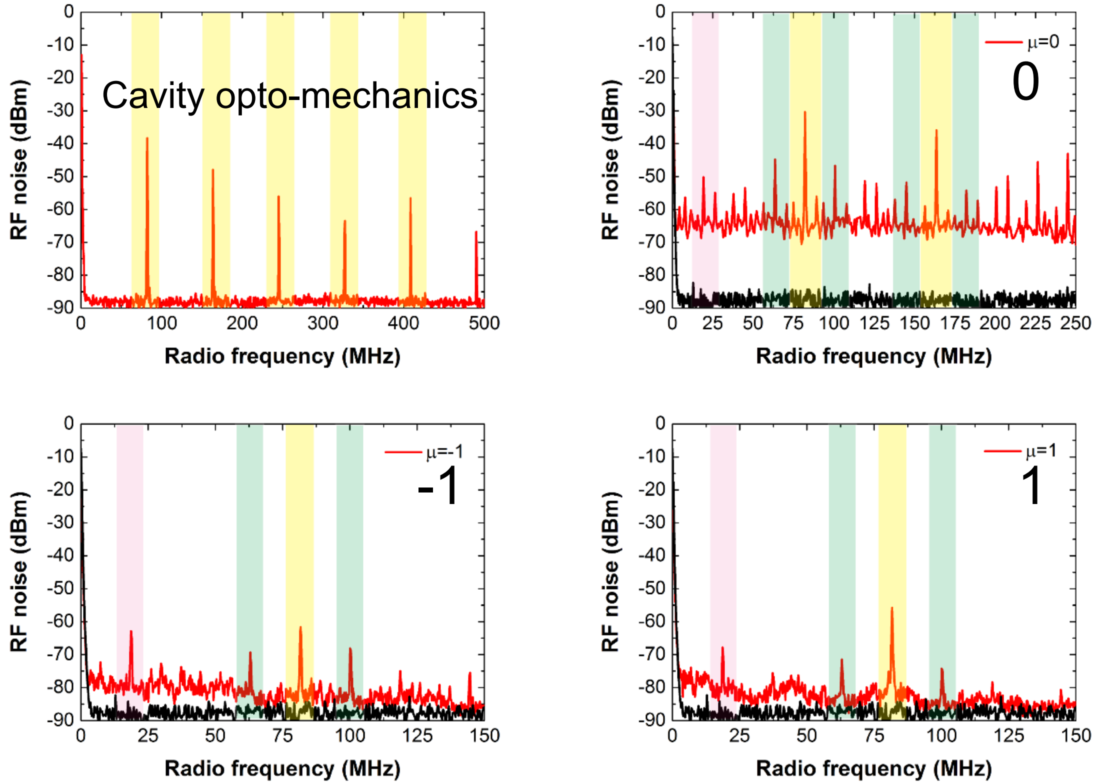
Effect of CW/CCW coupling: RF measurement

▶RF noise measurement



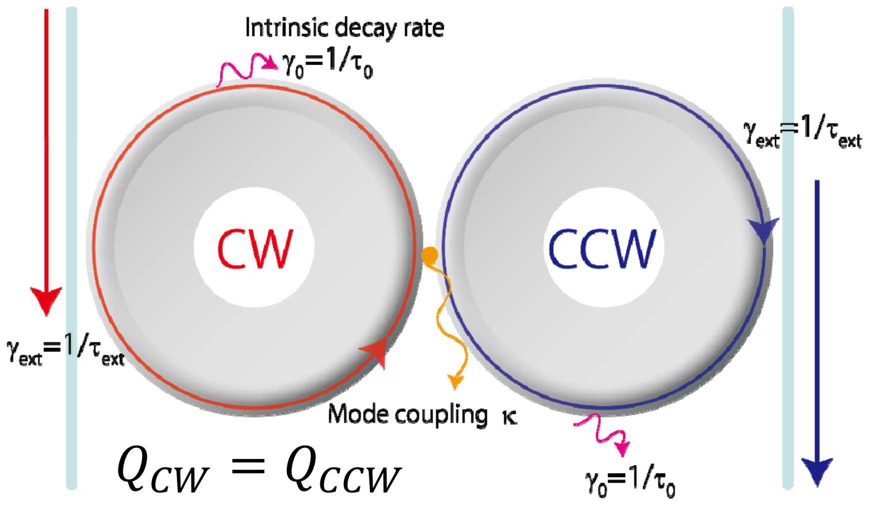
Each line is filtered out and the RF spectrum is measured

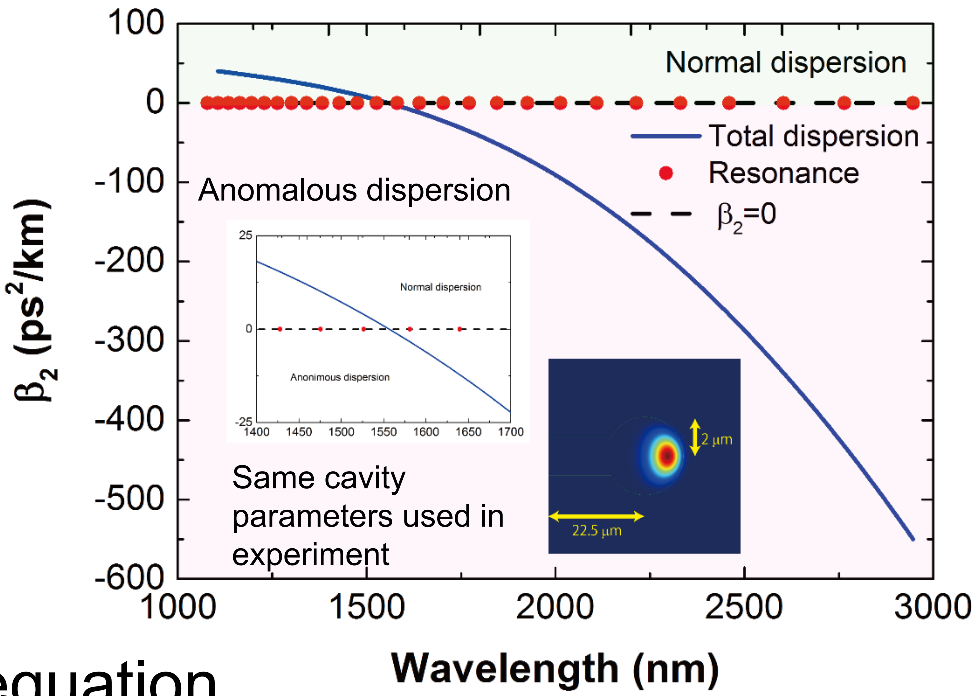
▶RF spectrum



No mode splitting effect is observed (Only the effect of cavity-opto mechanics is present)

Numerical analysis of CW/CCW coupling based on nonlinear CMT equations





▶Normalized nonlinear coupled mode equation

$$\frac{\partial \tilde{A}_\mu}{\partial t} = \frac{\gamma_{A_0}}{2}\left(-1+i\Omega_{A_\mu}\right)\tilde{A}_\mu + \frac{\gamma_{A_0}}{2}\cdot i\sum_{\alpha,\beta,\gamma}\delta_{\mu-(\alpha-\beta+\gamma)}\tilde{A}_\alpha\tilde{A}_\beta^*\tilde{A}_\gamma + i\delta_{\omega_{A_\mu}-\omega_{B_\theta}}\frac{\kappa_\theta}{2}\sqrt{\frac{g_A\gamma_{B_0}}{g_B\gamma_{A_0}}}\tilde{B}_\theta e^{i\{\omega_{A_\mu}-\omega_{B_\mu}-\mu(D_{1A}-D_{1B})\}t} + \frac{\gamma_{A_0}}{2}\delta_\mu f_0$$

$$\frac{\partial \tilde{B}_\mu}{\partial t} = \frac{\gamma_{B_0}}{2}\left(-1+i\Omega_{B_\mu}\right)\tilde{B}_\mu + \frac{\gamma_{B_0}}{2}\cdot i\sum_{\alpha,\beta,\gamma}\delta_{\mu-(\alpha-\beta+\gamma)}\tilde{B}_\alpha\tilde{B}_\beta^*\tilde{B}_\gamma + i\delta_{\omega_{B_\mu}-\omega_{A_\theta}}\frac{\kappa_\theta^*}{2}\sqrt{\frac{g_B\gamma_{A_0}}{g_A\gamma_{B_0}}}\tilde{A}_\theta e^{i\{\omega_{B_\mu}-\omega_{A_\mu}-\mu(D_{1B}-D_{1A})\}t}$$

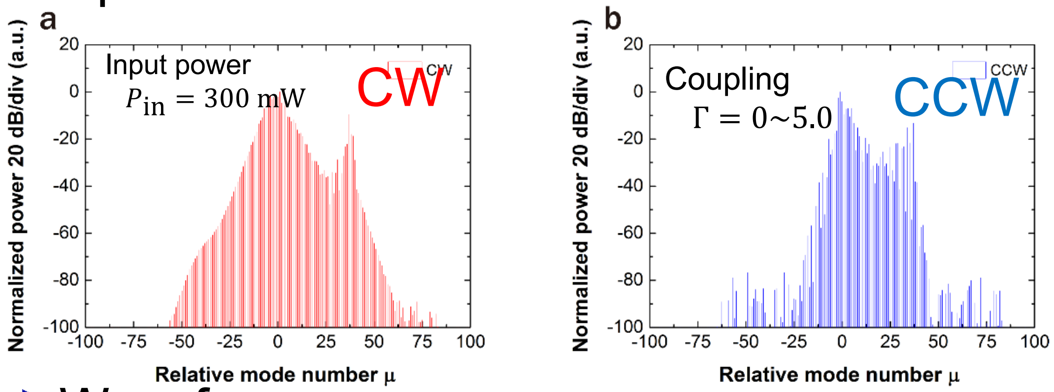
loss　FWM　CW/CCW mode coupling　input

$$\tilde{A}_\mu = \sqrt{2g_A/\gamma_{A_0}}\,A_\mu e^{i(\omega_{A_\mu}-\omega_{in}-\mu D_{1A})t},\ \Omega_{A_\mu} = 2\left(\omega_{A_\mu}-\omega_{in}-\mu D_{1A}\right)/\gamma_{A_0},\ f_0 = \sqrt{8g_A\gamma_{ext}/\kappa_{A_0}^3}\,A_{in}$$

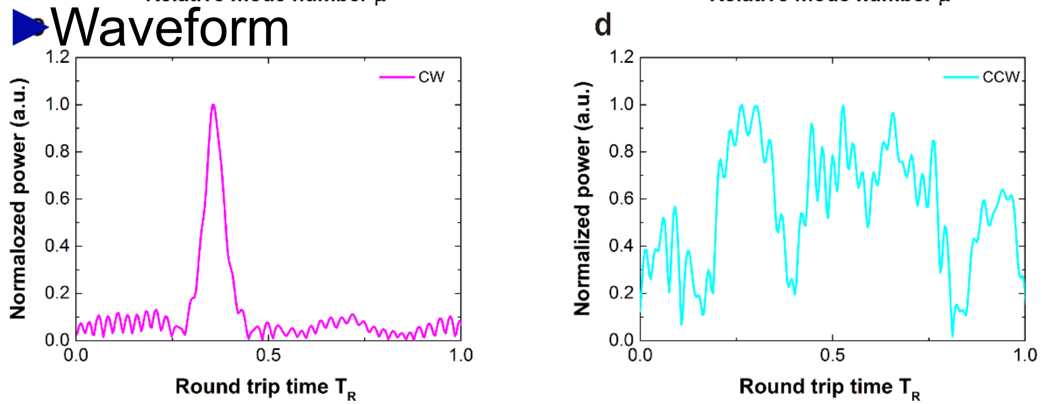
$$\tilde{B}_\mu = \sqrt{2g_B/\kappa_{B_0}}\,B_\mu e^{i(\omega_{B_\mu}-\omega_{in}-\mu D_{1B})t},\ \Omega_{B_\mu} = 2\left(\omega_{B_\mu}-\omega_{in}-\mu D_{1B}\right)/\gamma_{B_0}$$

Soliton formation in CW/CCW mode coupled system

▶Spectrum



▶Waveform

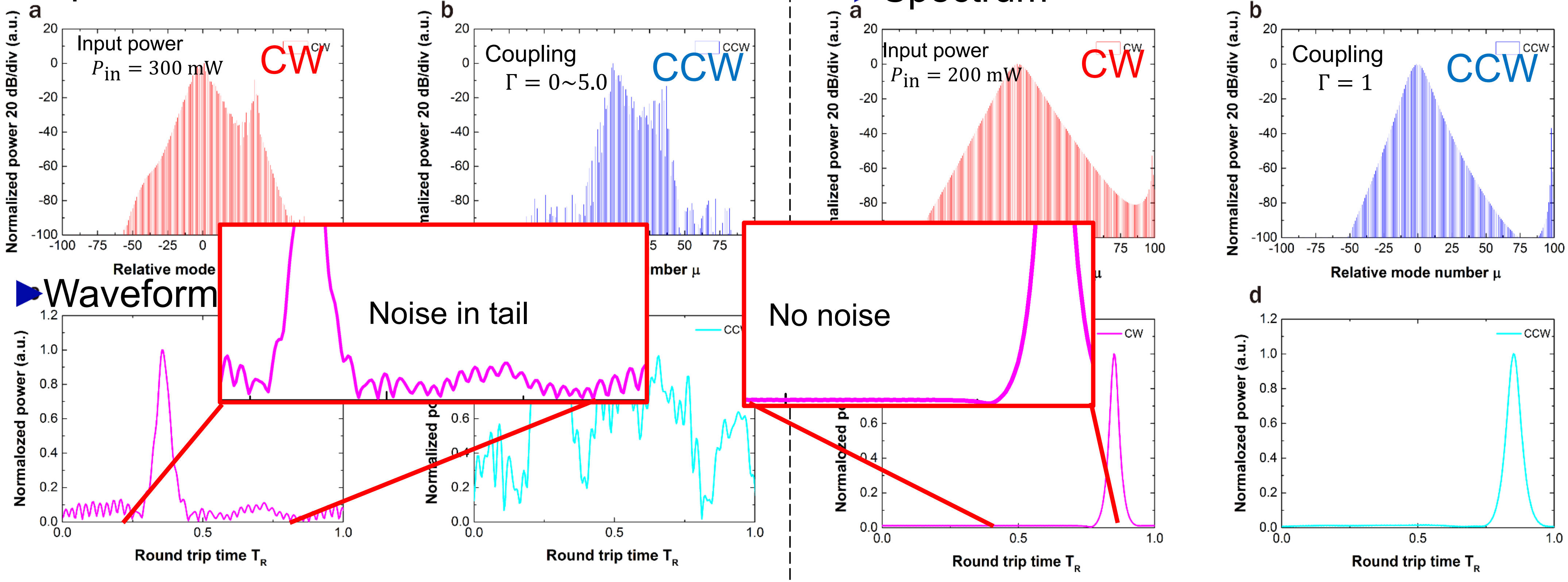


Soliton formed in CW only

Soliton formation in CW/CCW mode coupled system

▶Spectrum

▶Spectrum



Soliton formed in CW only

Soliton formed in CW/CCW modes

No noise appears when both CW/CCW modes are in soliton state

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Influence of XPM on Kerr comb: simulation

□ Lugiato-Lefever equations w/ TE/TM coupling

$$t_R \frac{\partial E_{TE}}{\partial r} = \left(-\alpha_{TE} - i\delta_{TE} + iL \sum_{k\geq 2} \frac{\beta_{k\_TE}}{k!} \left(i \frac{\partial}{\partial T} \right)^k \right) E_{TE} + i\gamma_{TE} L (|E_{TE}|^2 + PB|E_{TM}|^2) E_{TE} + \sqrt{\kappa_{TE}} S_{TE}$$

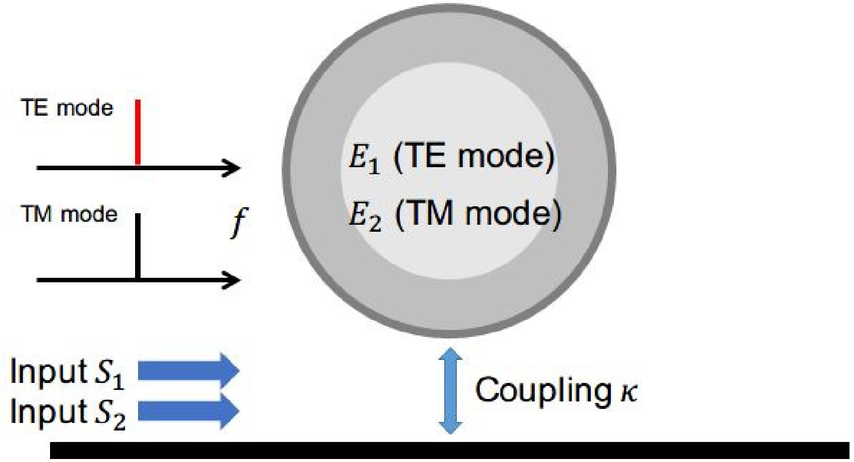
$$t_R \frac{\partial E_{TM}}{\partial r} = \left(-\alpha_{TM} - i\delta_{TM} + dLi \frac{\partial}{\partial T} + iL \sum_{k\geq 2} \frac{\beta_{k\_TM}}{k!} \left(i \frac{\partial}{\partial T} \right)^k \right) E_{TM} + i\gamma_{TM} L (|E_{TM}|^2 + PB|E_{TE}|^2) E_{TM} + \sqrt{\kappa_{TM}} S_{TM}$$

1. Mode overlapping(B) is perfect.
   → B should be changed. (0~1.0)

2. Group velocity mismatch(d) is zero.
   → d has to be calculated.

3. Detuning & cavity length(L) are same.
   → different detuning has to be set.

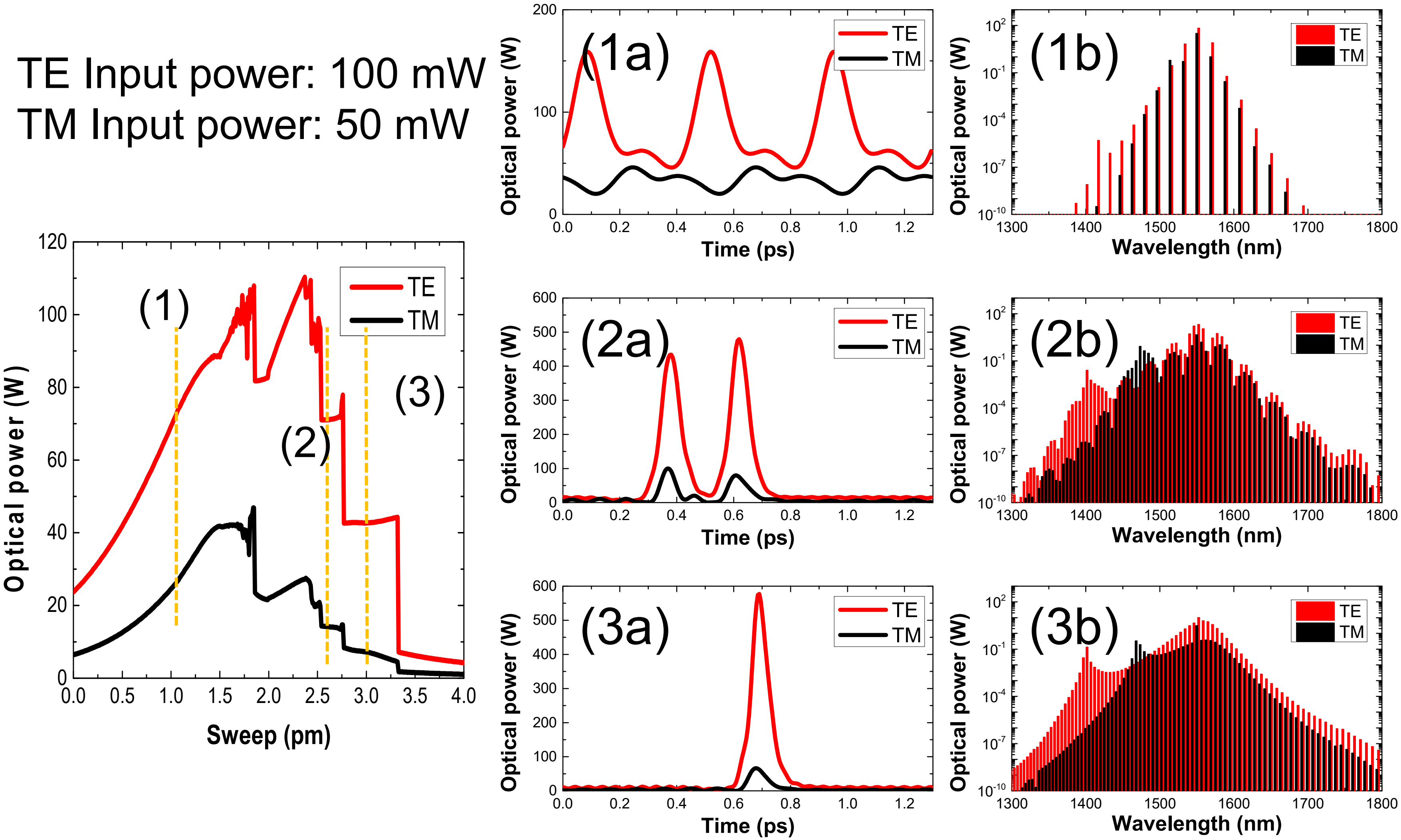
4. Linear coupling is zero.
   → EPFL members are interested in linear coupling condition.



Influence of XPM on Kerr comb: simulation

■ Dual comb generation from dual input…

TE Input power: 100 mW
TM Input power: 50 mW



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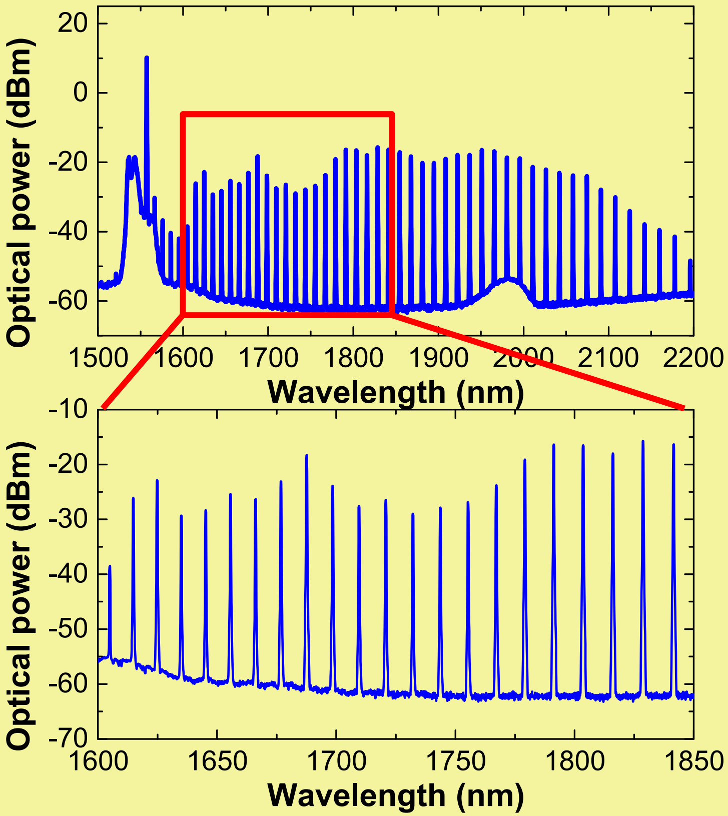
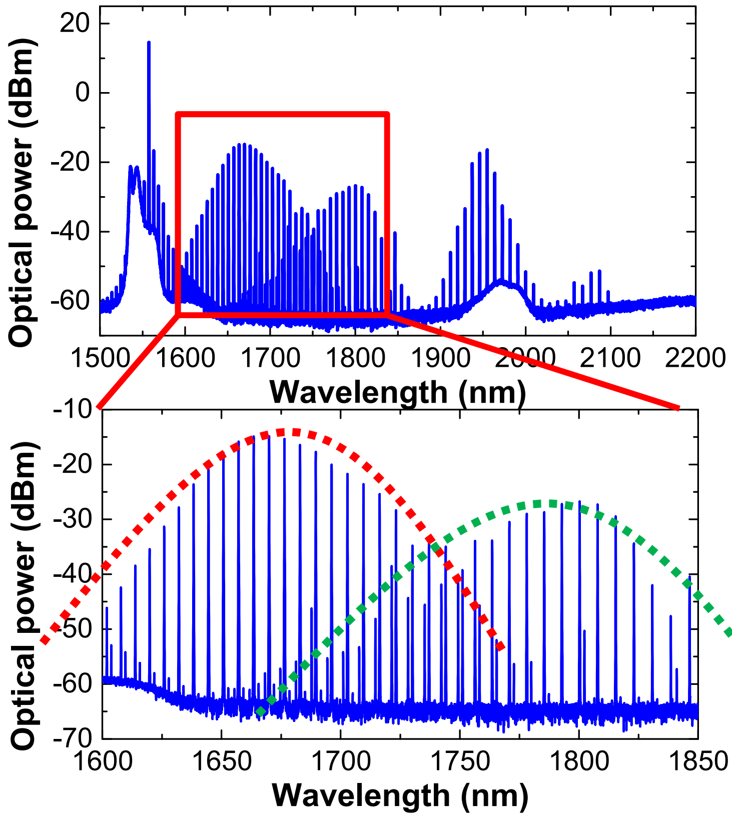
Different longitudinal mode coupling (experiment)

Resonator 1
- $\lambda_{\mathrm{pump}} = 1557.3$ nm

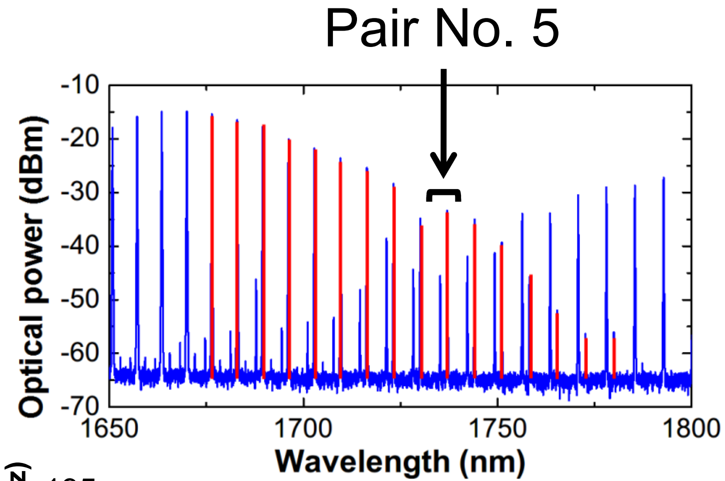


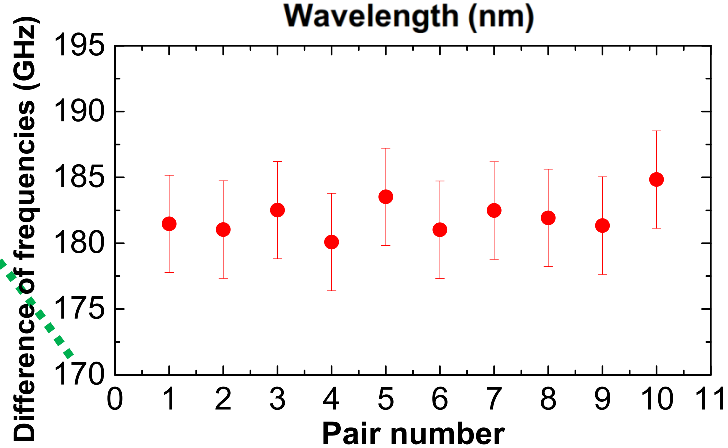
Same longitudinal mode family is generated

Resonator 2
- $\lambda_{\mathrm{pump}} = 1557.5$ nm



Different longitudinal mode family via Raman





Different mode is generated w/ 180 GHz offset

Coupled Lugiato-Lefever Equations

Lugiato-Lefever equations with Raman interaction

Mode 1 (pump mode)

$$t_R \frac{\partial E_p}{\partial t} = \left\{ -\frac{\alpha_p}{2} - \frac{\kappa_p}{2} - i\delta_p + iL \sum_{k\geq 2} \frac{\beta_p^{(k)}}{k!} \left(-i \frac{\partial}{\partial t} \right)^k \right\} E_p + iL\boldsymbol{N_p} + \sqrt{\kappa_p} S_{\text{in}}$$

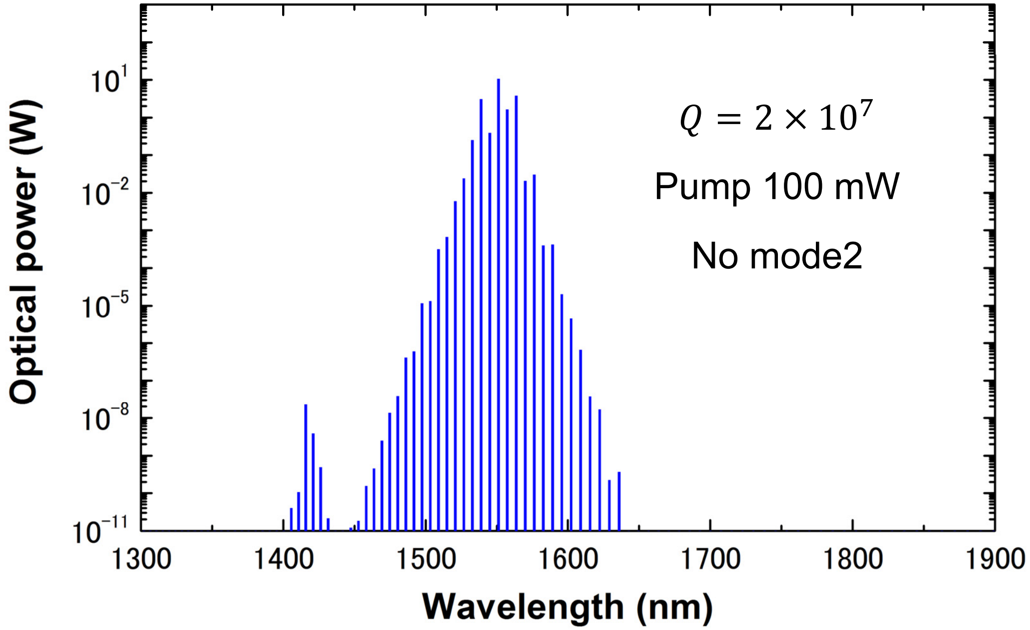
$$\boldsymbol{N_p} = (1 - f_R) \left(\underbrace{\gamma_p |E_p|^2}_{\text{SPM}} + \underbrace{2\Gamma_p |E_s|^2}_{\text{XPM}} \right) E_p + f_R \left\{ \begin{array}{c} \underbrace{\gamma_p E_p \int_{-\infty}^{\infty} h_R(t') |E_p(t-t')|^2 dt'}_{\text{SPM}} \\ + \underbrace{\Gamma_p E_p \int_{-\infty}^{\infty} h_R(t') |E_s(t-t')|^2 dt'}_{\text{XPM}} + \underbrace{\Gamma_p E_s \int_{-\infty}^{\infty} h_R(t') E_p(t-t') E_s^*(t-t') dt'}_{\text{Energy transfer due to Raman amplification}} \end{array} \right\}$$

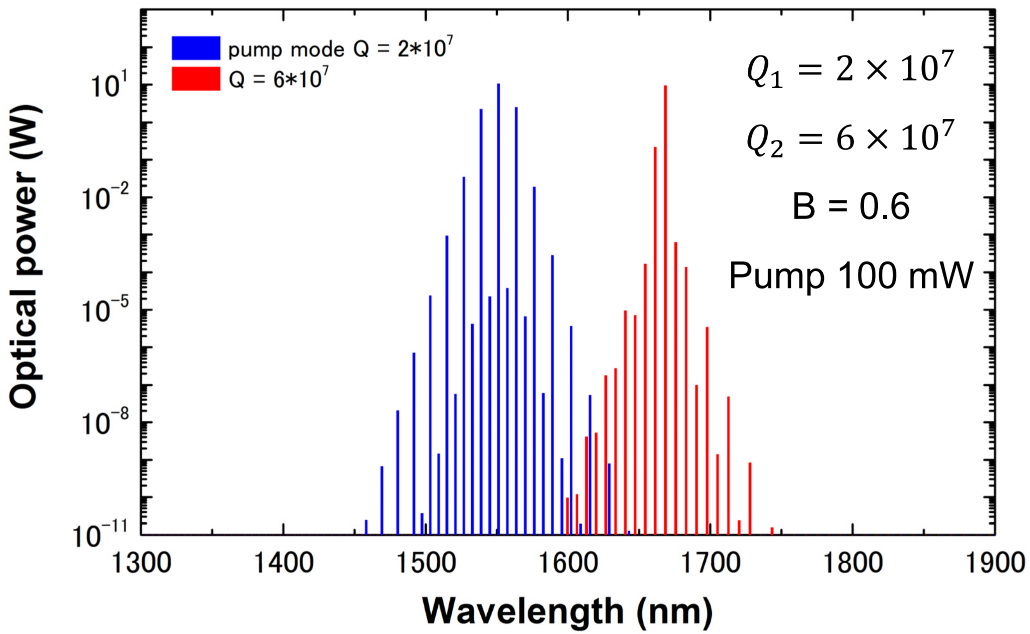
Mode 2 (Raman mode)

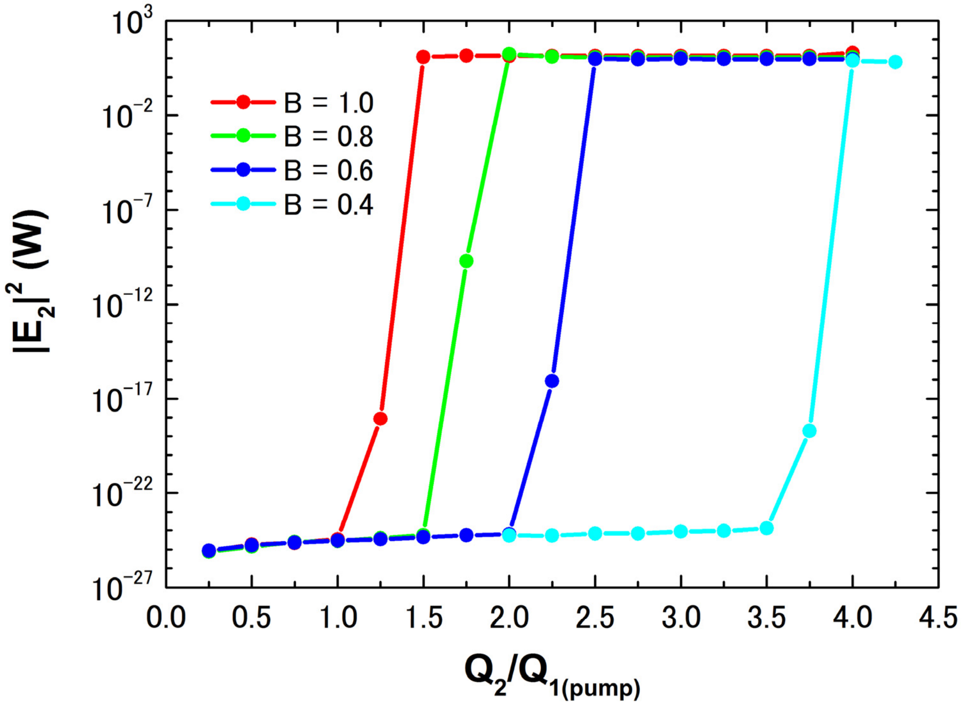
$$t_R \frac{\partial E_s}{\partial t} = \left\{ -\frac{\alpha_s}{2} - \frac{\kappa_s}{2} - iL(\beta_s^{(1)} - \beta_p^{(1)}) \left(-i \frac{\partial}{\partial t} \right) + iL \sum_{k\geq 2} \frac{\beta_s^{(k)}}{k!} \left(-i \frac{\partial}{\partial t} \right)^k \right\} E_s + iL\boldsymbol{N_s}$$

$$\boldsymbol{N_s} = (1 - f_R) \left(\gamma_s |E_s|^2 + 2\Gamma_s |E_p|^2 \right) E_s + f_R \left\{ \begin{array}{c} \gamma_s E_s \int_{-\infty}^{\infty} h_R(t') |E_s(t-t')|^2 dt' \\ + \Gamma_s E_s \int_{-\infty}^{\infty} h_R(t') |E_p(t-t')|^2 dt' + \Gamma_s E_p \int_{-\infty}^{\infty} h_R(t') E_s(t-t') E_p^*(t-t') dt' \end{array} \right\}$$

Spectrum and intra-cavity power







$$Q_{1_{pump}} = 2 \times 10^7$$

$$Q_e = 2 \times 10^8$$

Pump power = 100 mW

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Summary

We investigate the effect of mode coupling

1. Mode crossing: Mode crossing needed to initiate dark soliton
2. CW/CCW mode coupling: Small noise present from CCW mode scattering, but this can be suppressed by choosing right parameters.
3. TM/TE mode coupling: Injection locking occurs which is a powerful method to achieve dual comb generation
4. Nonlinear coupling between modes via Raman: Mode coupling occurs from a low to high Q

Mode coupling enable rich function

Acknowledgement

The team

Mr. Takumi Kato (Kerr comb)
Mr. Ryo Suzuki (cavity opto-mechanics)
Mr. Wataru Yoshiki (CW/CCW coupling)
Mr. Shun Fujii (CW/CCW mode coupled FWM)
Mr. Akitoshi C.-Jinnai (THG generation)
Mr. Tomoya Kobatake (LL calculation)
Mr. Yusuke Okabe (nonlinear CMT calculation)
Ms. Misako Kobayashi (packaging)

Collaborators

Prof. Y. Kakinuma (Keio Univ.)

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総務省
MIC Ministry of Internal Affairs and Communications

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