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Noninear Coupled Mode Equations for Kerr Comb Generation in Coupled Microcavity System

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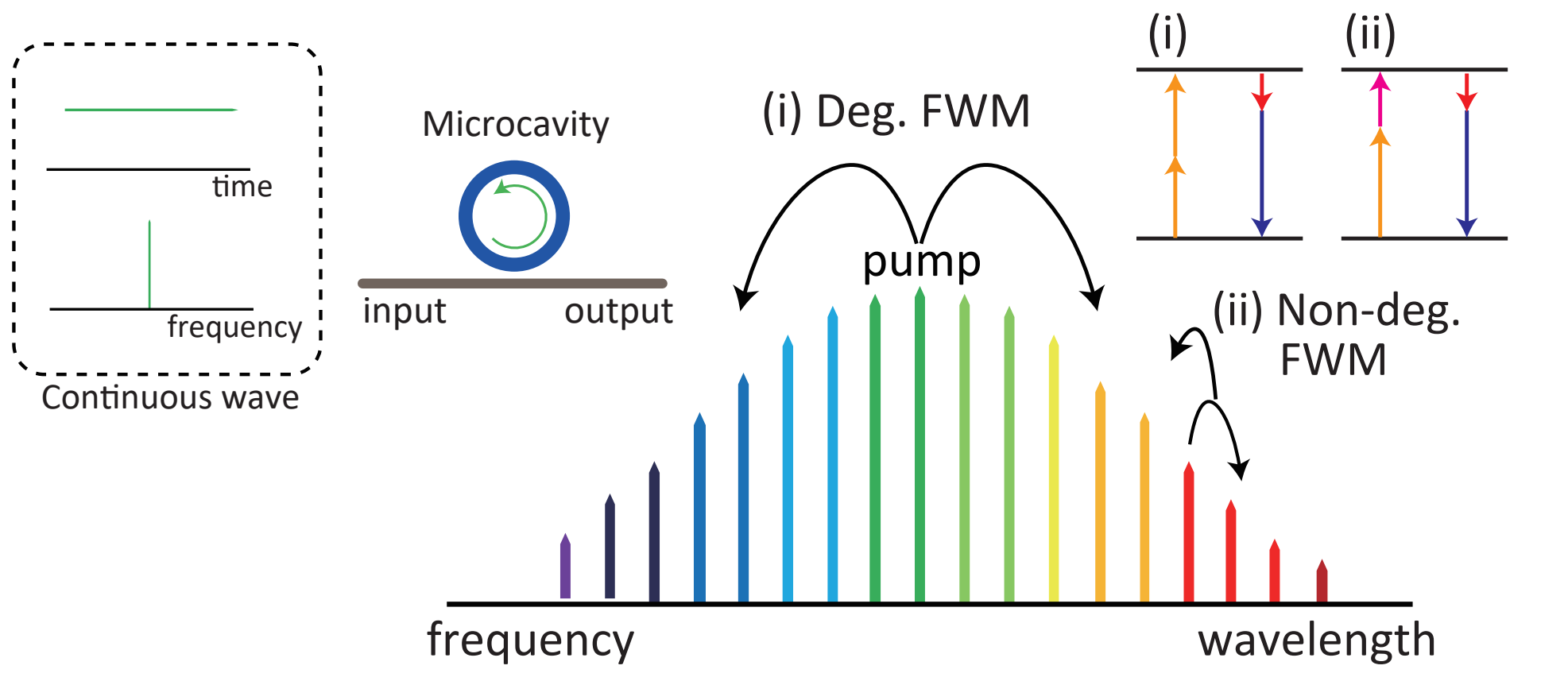
Abstract

Kerr combs, which are generated fom microcavities, have been intensively investigated for a variety of applications. The model of Kerr comb formation has been developed using two approaches: a nonlinear coupled mode equation (NCME) and a Lugiato-Lefever equation (LLE). In this work, we performed a rigorous numerical simulation based on NCMEs of normal dispersion Kerr comb generation that is possible by employing mode coupling between two different mode families.

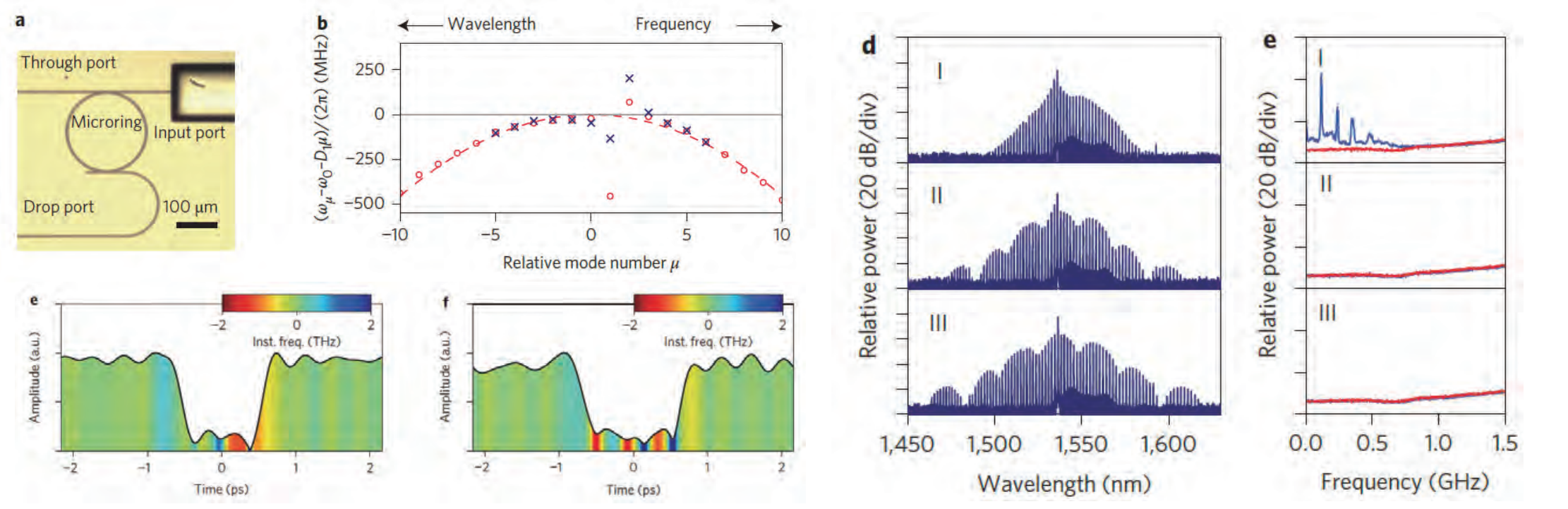
Background

1. Kerr comb generation via four-wave mixing



Mode-locked Kerr comb generated by continuous wave (CW) pump via degenerate and non-degenerate FWM.

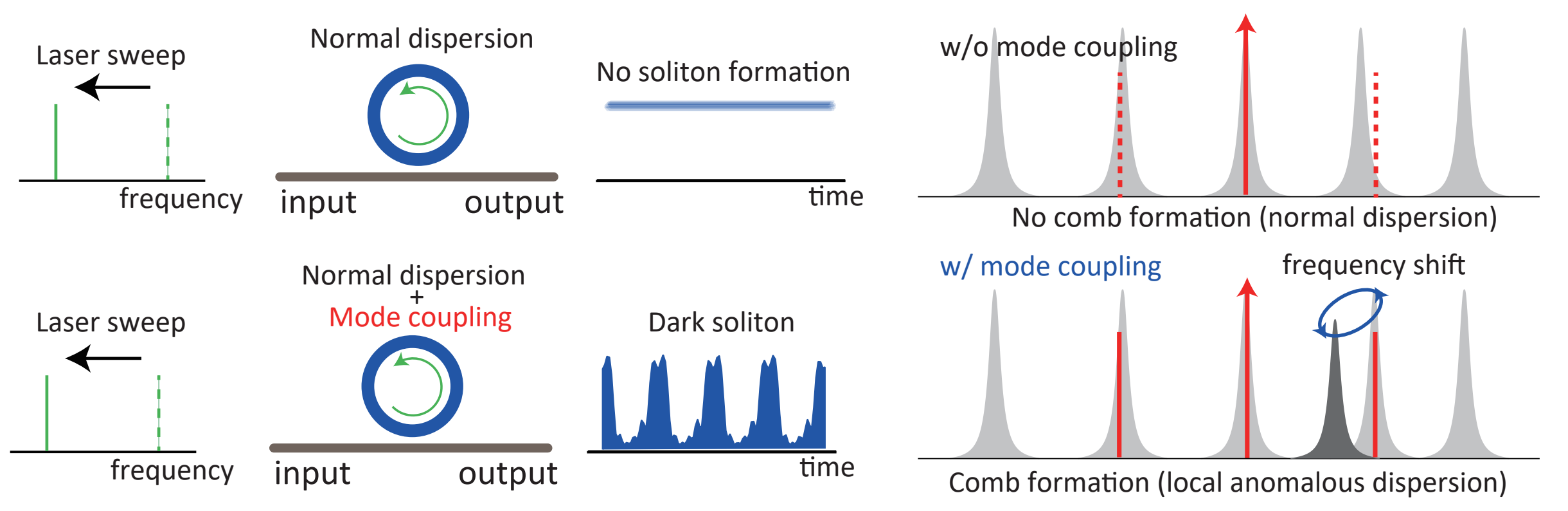
2.（Previous work）Experimental demonstrations



X. Xue et al. Nat. Photonics 9, 594-600 (2015).

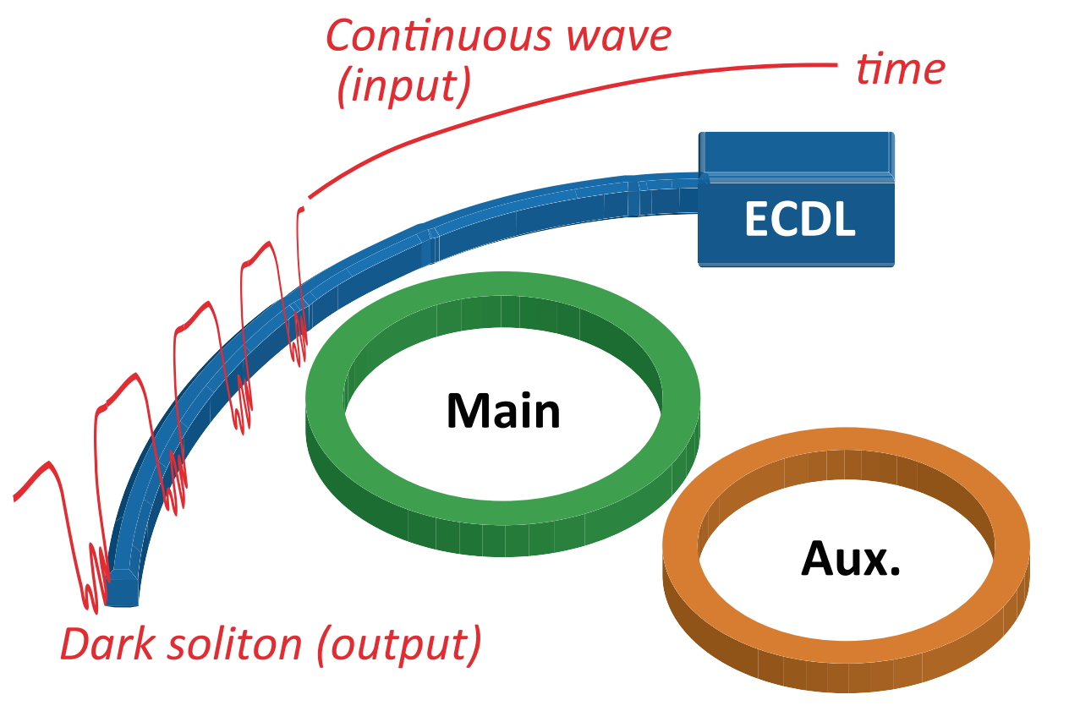
Mode-locked dark pulse formation demonstrated in the experiment but little simulation studies that include mode coupling effect.

3. Normal dispersion comb induced by mode coupling



Effective frequency shift by mode coupling assists the phase-matching and intial comb sidebands in the coupled resonance.

Numerical modeling



$$\frac{da_\mu}{dt} = -\left[\frac{\gamma_A}{2} + i(\omega_{\mu A} - \omega_p - \mu D_1)\right] a_\mu + i g_A \sum_{j,k} a_j a_k a^*_{j+k-\mu} + i\frac{\kappa}{2} b_\mu + f\delta_\mu$$

$$\frac{db_\mu}{dt} = -\left[\frac{\gamma_B}{2} + i(\omega_{\mu B} - \omega_p - \mu D_1)\right] b_\mu + i g_B \sum_{j,k} b_j b_k b^*_{j+k-\mu} + i\frac{\kappa}{2} a_\mu$$

Mode amplitude — Decay rate — Detuning/dispersion — Kerr effect SPM/XPM/FWM — Mode coupling — Input

Discrete resonance frequencies

$$\omega_\mu = \omega_0 + D_1\mu + \frac{1}{2}D_2\mu^2$$

Kerr coefficient

$$g = \frac{\hbar\omega_0^2 c n_2}{n_0^2 V_{\mathrm{eff}}}$$

Pump term

$$f = \sqrt{\frac{\gamma_{\mathrm{ext}} P_{\mathrm{in}}}{\hbar\omega_0}}$$

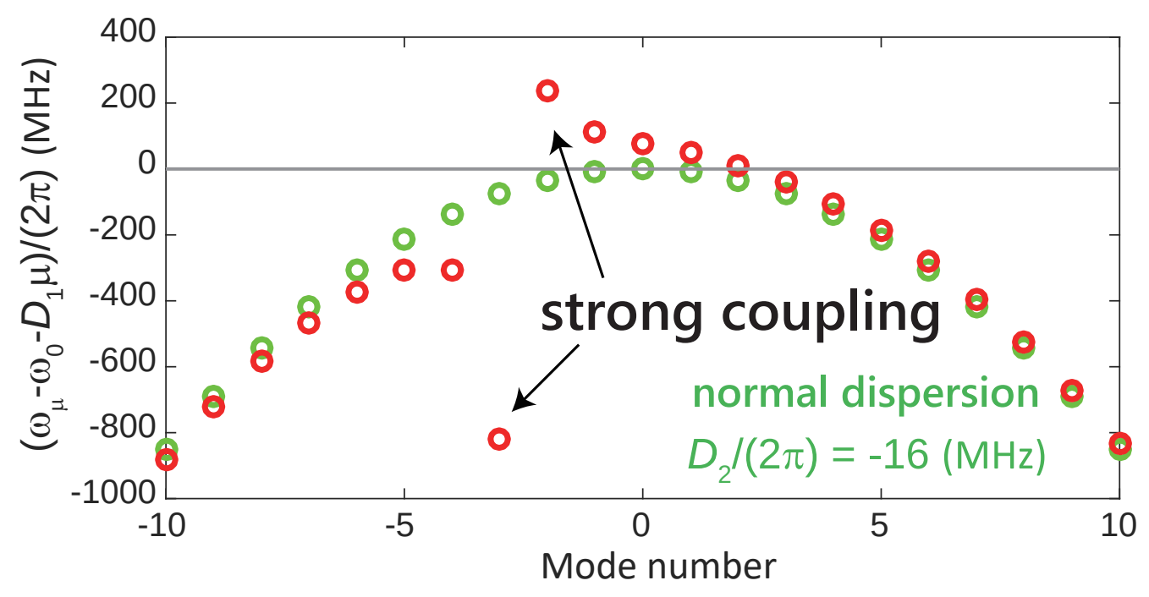
center freq.	ω_0 (rad · Hz)
cavity FSR	D_1 (rad · Hz)
dispersion	D_2 (rad · Hz)
coupling	κ (rad · Hz)
decay rate	$\gamma = \omega_0/Q$

Simulation results

Simulation parameters (SiN microring)

Main mode (A)	Aux. mode (B)
$\omega_{0A}/2\pi = 191.9$ (THz)	$\omega_{0B}/2\pi = 191.936$ (THz)
$D_{1A}/2\pi = 378$ (GHz)	$D_{1B}/2\pi = 391$ (GHz)
$D_{2A}/2\pi = -16$ (MHz)	$D_{2B}/2\pi = -17$ (MHz)
$Q_A = 7.5 \times 10^5$	$Q_B = 3.7 \times 10^5$
$P_{\mathrm{in}} = 500$ (mW)	$Q_{\mathrm{ext}} = 3.5 \times 10^6$
$\kappa/2\pi = 3.34$ (GHz)	$A_{\mathrm{eff}} = 1.10$ (μm^2)

Dispersion affected by mode coupling



Resonance assymetry factor for initial comb

D₂<0 (normal disp.)→No comb formation

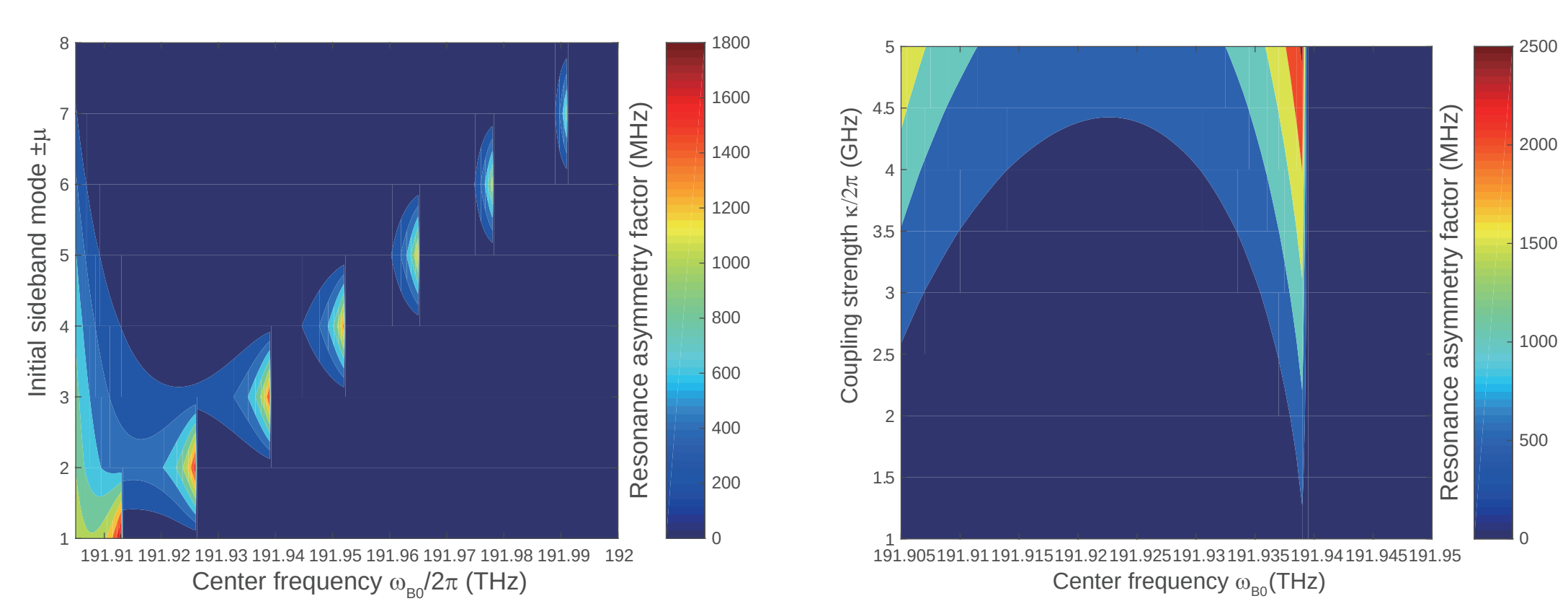
$$\Delta^2\omega = \omega_\mu - \omega_0 - (\omega_0 - \omega_{-\mu}) = D_2\mu^2 < 0$$

D₂>0 (anomalous disp.)→Initial comb sidebands from ±μ

$$\Delta^2\omega = \omega_\mu - \omega_0 - (\omega_0 - \omega_{-\mu}) = D_2\mu^2 > 0$$

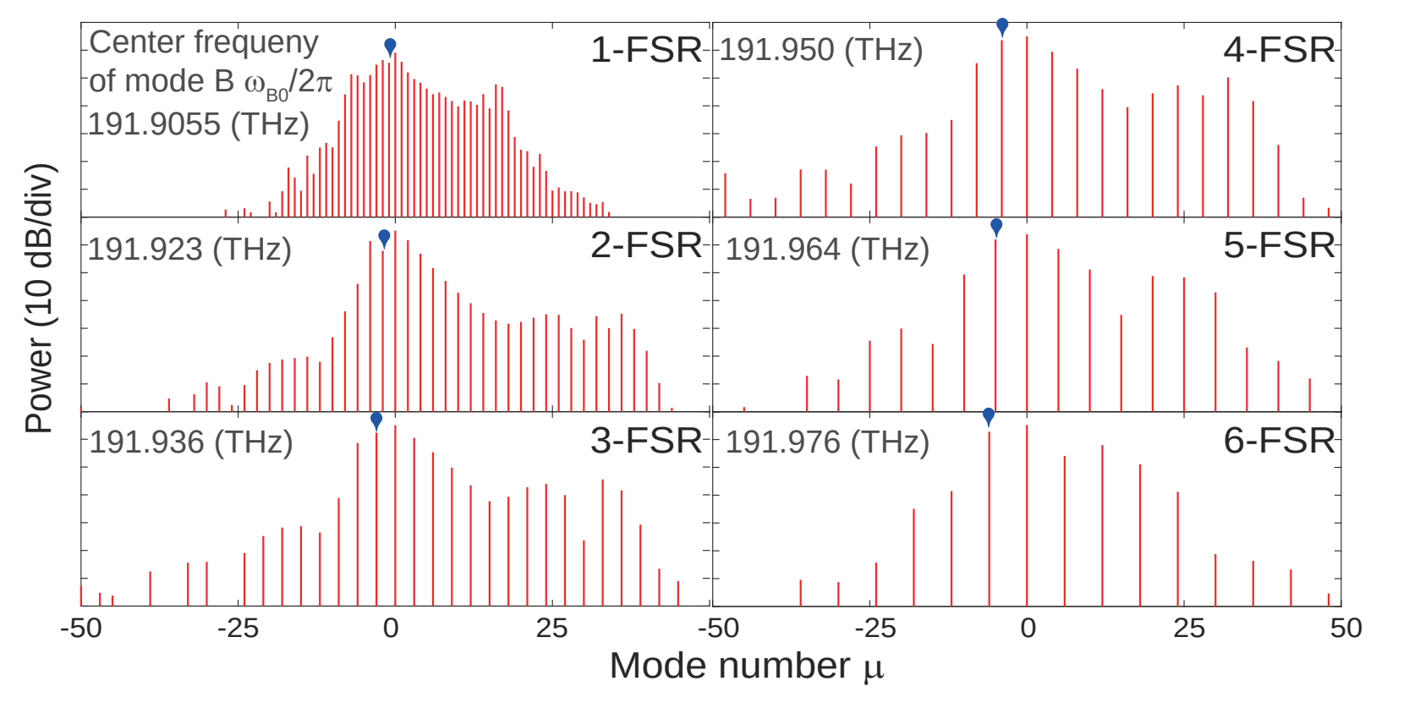
Theoretical analysis of phase-matching condition

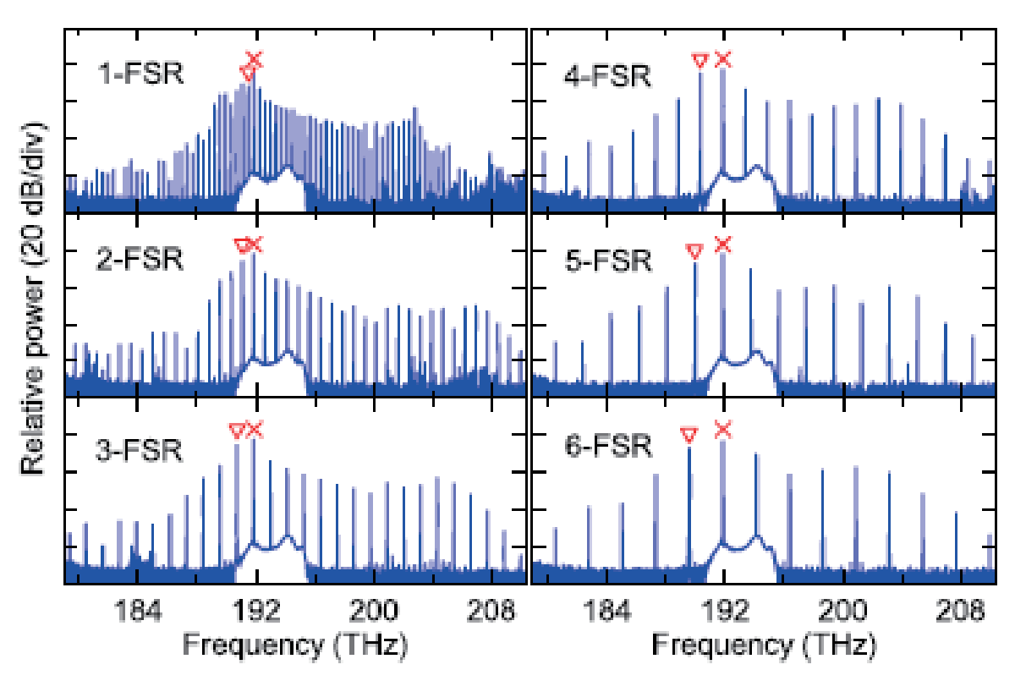
Mode number of initial sideband — Coupling strength in 3-FSR comb



FSR selectable comb (center frequency of mode B changed)

Simulation results — (Previous work) Experimental results

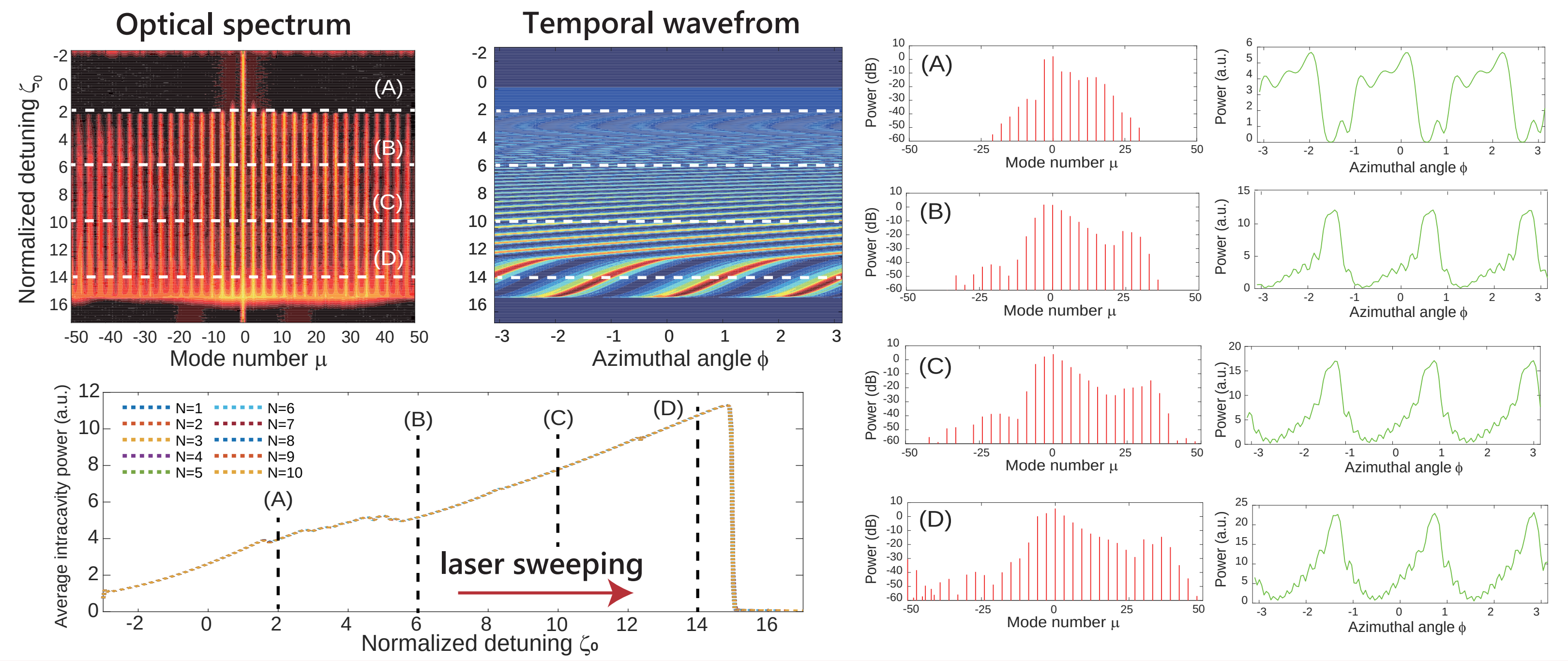




X. Xue et al. Laser Photonics Rev. 9, L23-L28 (2015).

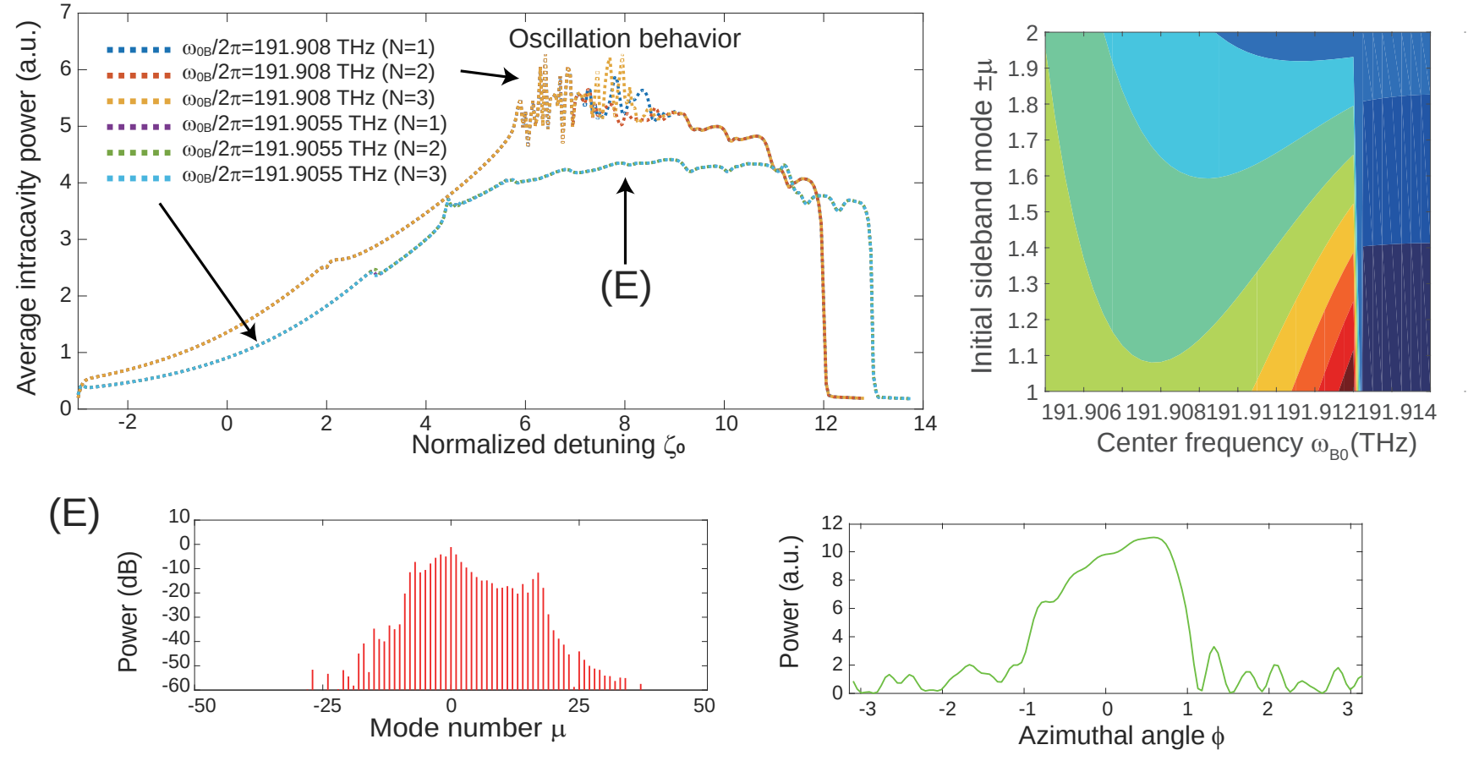
3-FSR Kerr comb and dark soliton simulation

Optical spectrum — Temporal wavefrom



Oscillating behavior of intracavity power

1-FSR Kerr comb simulation



Observed oscillating behavior of the trajectory of intracavity power depending on center frequency of mode B.
New findings of stable/unstable operating conditions but more investigation of the reason required!

Conclusion

We studied Kerr comb generation with nonlinear coupled mode equations by taking rigorous mode coupling model into account. A theoretical analysis of the phase matching condition allowed us to simulate FSR selectable comb generation more easily and rigorously. This modeling approach will be a powerful tool for assisting future work in terms of dispersion engineering for Kerr comb generation and frequency tuning for deterministic mode-locked comb generation.

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