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Soliton trapping in a Kerr microresonator with orthogonally polarized dual-pumping

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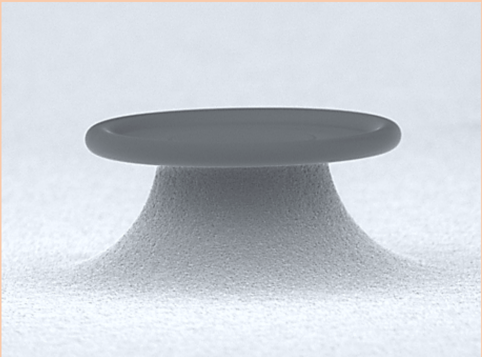
Microcombs

Laser light having a comb-like spectrum, which is generated from a microresonator.

“Microcombs” or “Kerr combs”

Microresonator



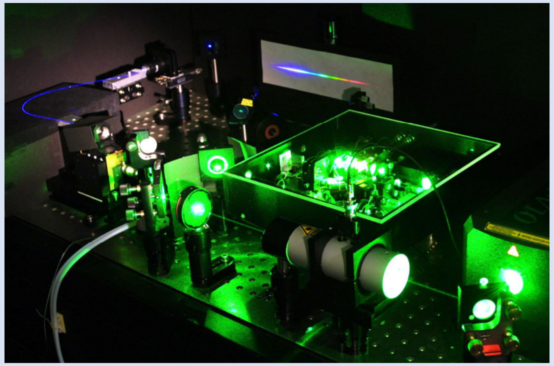
- Compact size
- Low consumption energy
- Large mode spacing (f_{rep}~ 10-1000 GHz)

Applications

- Optical communications
- Dual-comb spectroscopy
- Dual-comb LiDAR
- Microwave oscillators
- Optical frequency synthesizers

“Frequency combs”

Ti:Sapphire laser



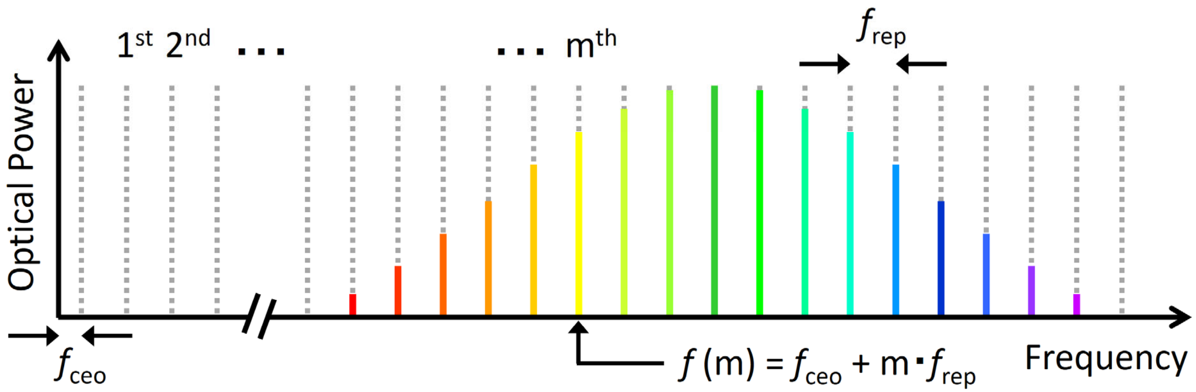
http://www.mpq.mpg.de/~haensch/comb/index.html

Fiber laser

https://www.aist.go.jp/index_ja.html

- Small mode spacing (f_{rep}~ 0.01-10 GHz)

Comb spectrum

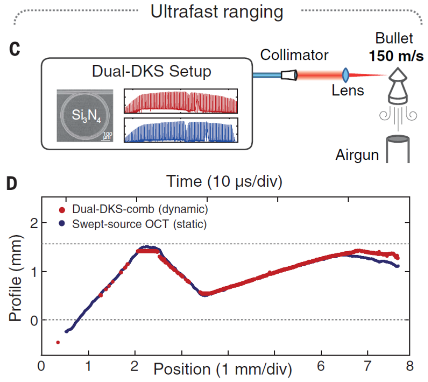


Dual-comb applications

Dual-comb applications: scan rate ⇔ difference of repetition frequencies

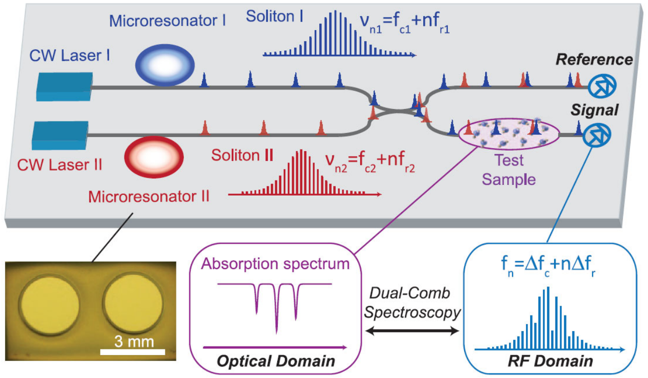
Microcombs have a potential to achieve fast scan rate due to high repetition frequencies

LiDAR



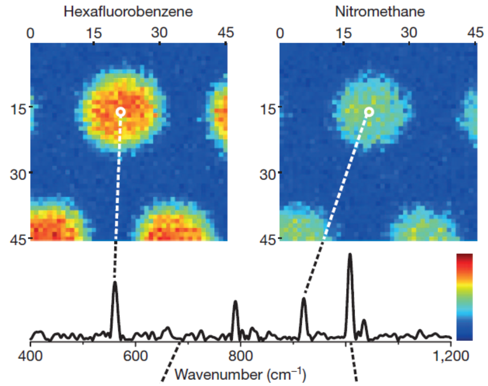
Science 359, 887-891 (2018)

Spectroscopy



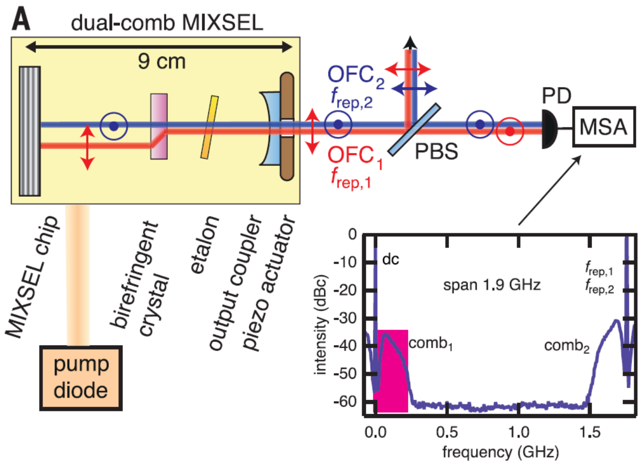
Science 354, 600-603 (2016)

CARS



Nature 502, 355-358 (2013)

Dual-comb generation in a single resonator

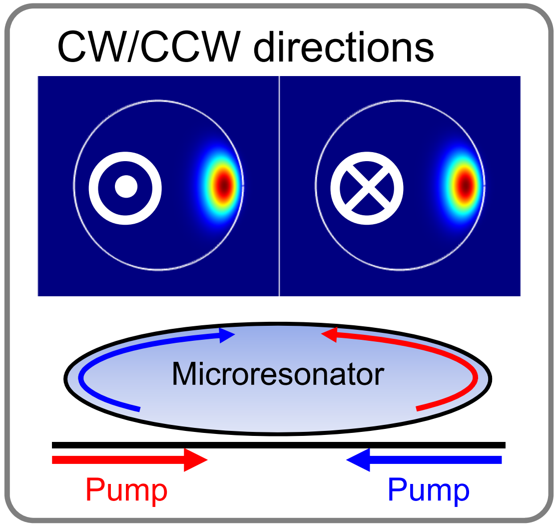


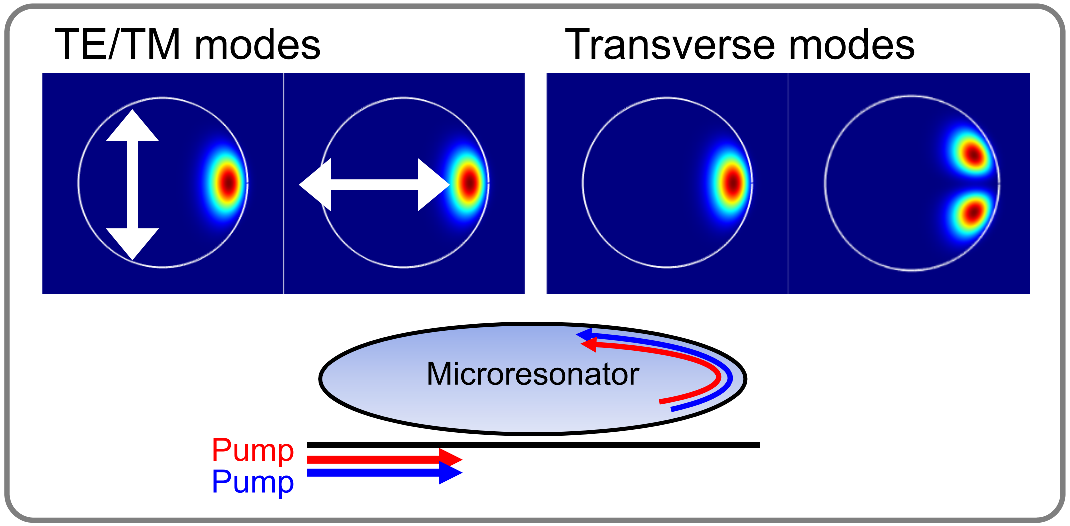
Science 356, 1164-1168 (2017)

Advantages

- Simple setup
- Both combs share the same resonator (common mechanical vibrations) and the feedback loops, which lead to mutual coherence.

Dual-comb generation in a single microresonator



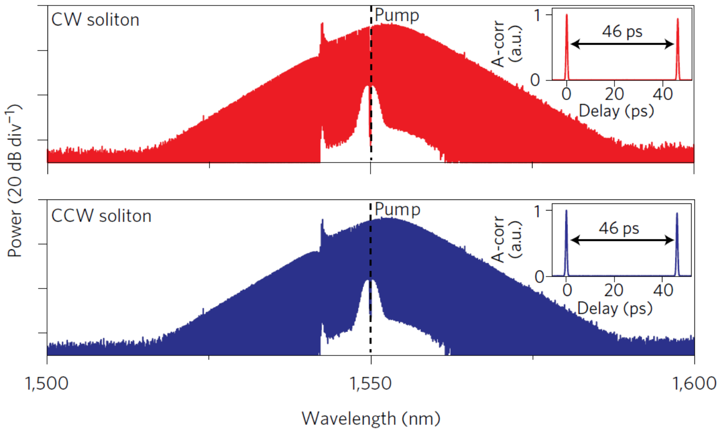


☺ Simple control of pump frequencies
☹ Small repetition rate difference

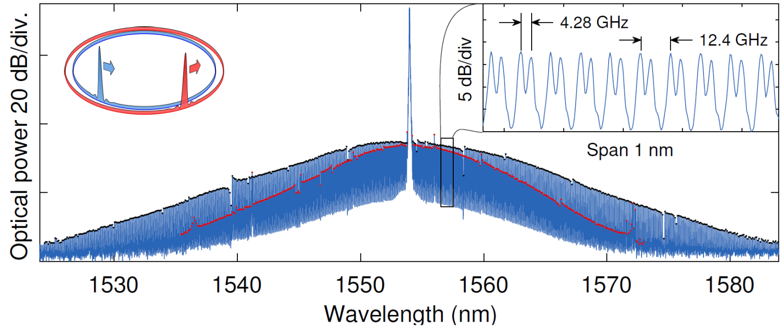
☹ Complex control of pump frequencies
☺ Large repetition rate difference

Recently, some experimental demonstrations have been reported.

CW/CCW directions



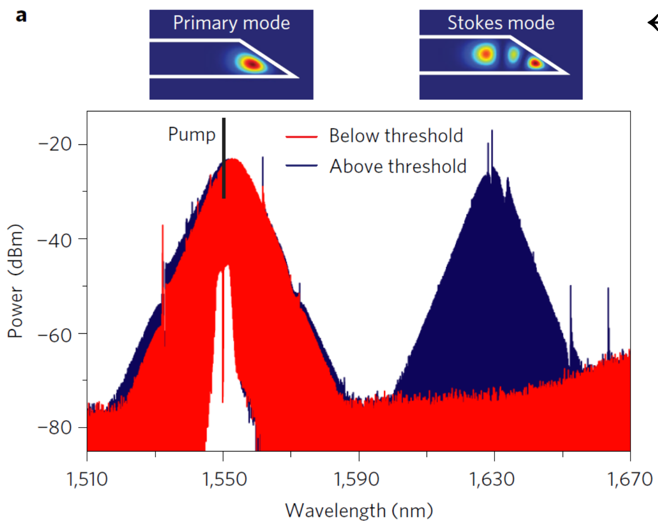
Transverse modes



Left: Nat. Photonics 11, 560-564 (2017)
Right: arXiv preprint arXiv:1804.03706 (2018)

Motivation of this study

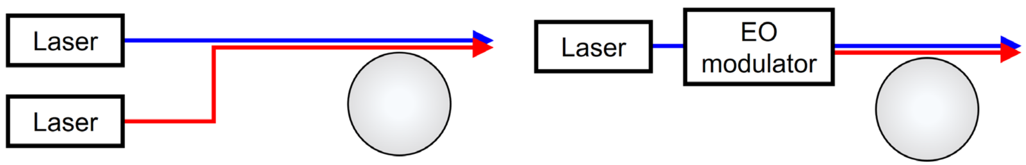
Interaction between two solitons in a microresonator has not been well understood. Here we focus on soliton trapping between orthogonally polarized solitons.



Nat. Phys. 13, 53-57 (2017)

← <u>In previous research,</u>
soliton trapping has been observed experimentally via Raman effects with single-pumping.

<u>In this work,</u>
we consider a system where two solitons are excited with dual-pumping having orthogonally polarizations.



In this work,

- Develop a simulation model based on coupled Lugiato-Lefever equations (LLEs), taking cross-phase modulation (XPM) and repetition difference terms into account.
- Calculate with generalized parameters to reveal trapping conditions
- Perform analysis of coupled solitons solutions.

Simulation model

Coupled Lugiato-Lefever equations (LLEs)

$$\frac{\partial a}{\partial t} = -\frac{\kappa_{(a)}}{2}a + i\Delta\omega_{0(a)}a + i\frac{D_{2(a)}}{2}\frac{\partial^2 a}{\partial\phi^2} + ig_{(a)}(|a|^2 + \sigma|b|^2)a + \sqrt{\kappa_{c(a)}}s_{\mathrm{in}(a)} + \frac{\Delta D_1}{2}\frac{\partial a}{\partial\phi}$$

$$\frac{\partial b}{\partial t} = -\frac{\kappa_{(b)}}{2}b + i\Delta\omega_{0(b)}b + i\frac{D_{2(b)}}{2}\frac{\partial^2 b}{\partial\phi^2} + ig_{(b)}(|b|^2 + \sigma|a|^2)b + \sqrt{\kappa_{c(b)}}s_{\mathrm{in}(b)} - \frac{\Delta D_1}{2}\frac{\partial b}{\partial\phi}$$

(loss) (detuning) (dispersion) (Kerr effects) (input) (repetition difference)

t: time, ϕ: angular coordinate, a, b: internal fields, κ: resonator loss, $\Delta\omega_0$: pump detuning, D_2: second order dispersion, g: nonlinear coefficient, σ: XPM coefficient (σ = 2/3 for orthogonally polarizations), κ_c: coupling rate, s_{in}: input field, ΔD_1: FSR (repetition frequency) difference

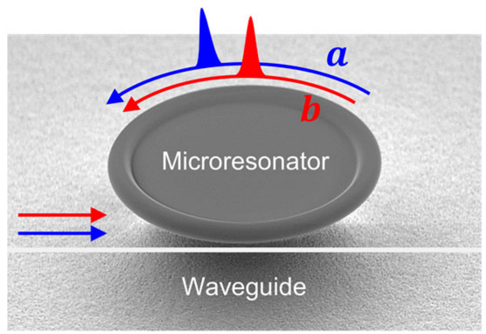
Dimensionless coupled LLEs

(Assuming $\kappa = \kappa_{(a)} = \kappa_{(b)}$, $g = g_{(a)} = g_{(b)}$)

$$\frac{\partial u}{\partial\tau} = -(1 + i\alpha_{(u)})u + i\beta_{(u)}\frac{\partial^2 u}{\partial\phi^2} + i(|u|^2 + \sigma|v|^2)u + F_{(u)} + \gamma\frac{\partial u}{\partial\phi}$$

$$\frac{\partial v}{\partial\tau} = -(1 + i\alpha_{(v)})v + i\beta_{(v)}\frac{\partial^2 v}{\partial\phi^2} + i(|v|^2 + \sigma|u|^2)v + F_{(v)} - \gamma\frac{\partial v}{\partial\phi}$$

$\tau = \frac{1}{2}\kappa t$, $u = \sqrt{\frac{2g}{\kappa}}a$, $v = \sqrt{\frac{2g}{\kappa}}b$, $\alpha_{(*)} = -\frac{2\Delta\omega_{0(*)}}{\kappa}$, $\beta_{(*)} = \frac{D_{2(*)}}{\kappa}$, $\gamma = \frac{\Delta D_1}{\kappa}$, $F_{(*)} = \frac{2}{\kappa}\sqrt{\frac{2g\kappa_{c(*)}}{\kappa}}s_{\mathrm{in}(*)}$



Relations, α: detuning, β: second order dispersion, γ: repetition difference, F: input

Soliton trapping with dimensionless coupled LLEs

$$\frac{\partial u}{\partial \tau} = -\left(1 + i\alpha_{(u)}\right)u + i\beta_{(u)}\frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma|v|^2)u + F_{(u)} + \gamma\frac{\partial u}{\partial \phi}$$

$$\frac{\partial v}{\partial \tau} = -\left(1 + i\alpha_{(v)}\right)v + i\beta_{(v)}\frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + \sigma|u|^2)v + F_{(v)} - \gamma\frac{\partial v}{\partial \phi}$$

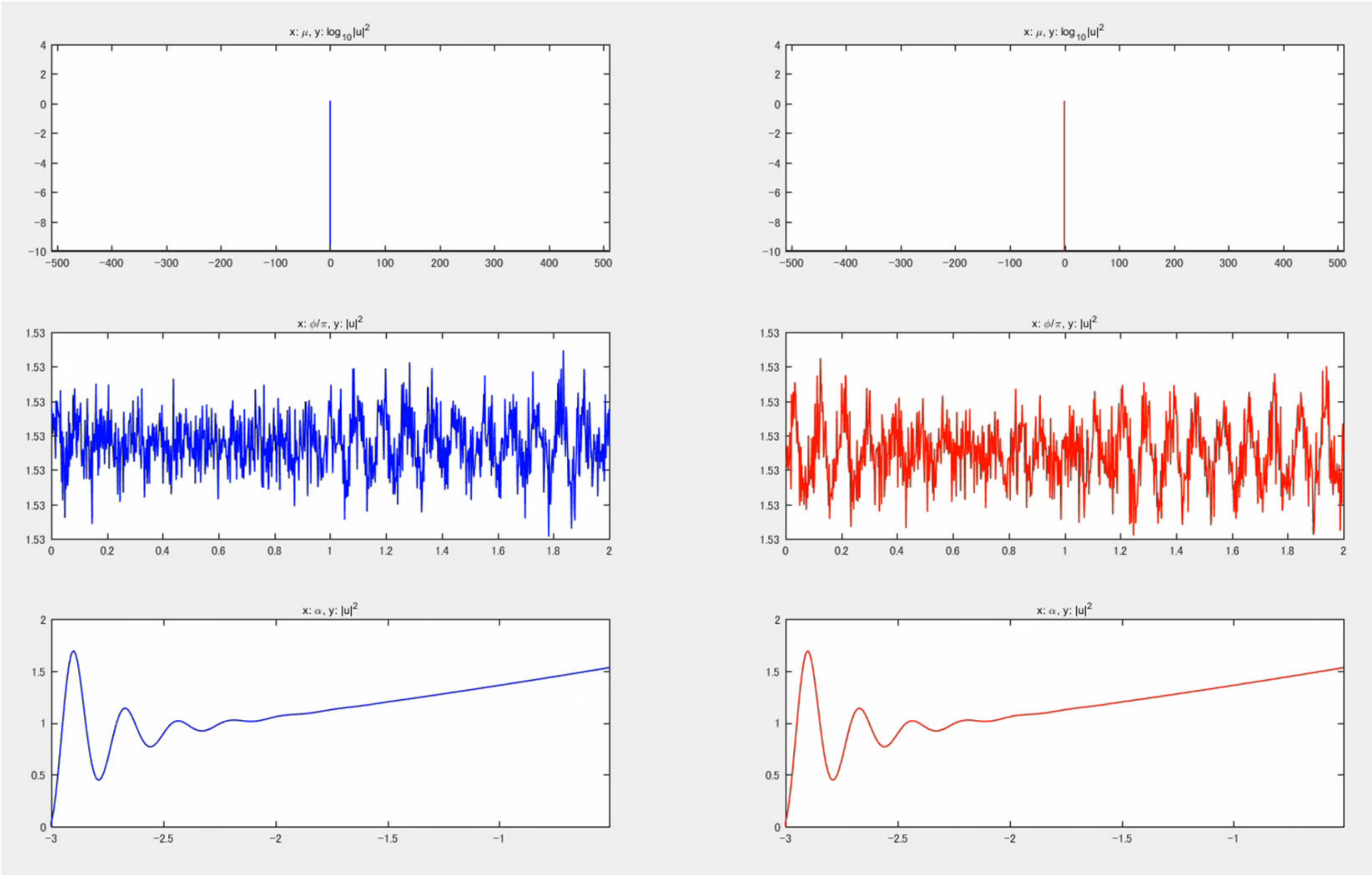
$\beta_{(*)} = 0.01, \gamma = 0.3, F_{(*)} = 4$
α is scanned

Spectrum

Waveform

Moving at different speeds:
Micorocombs propagate
at different group velocities

Intracavity
power



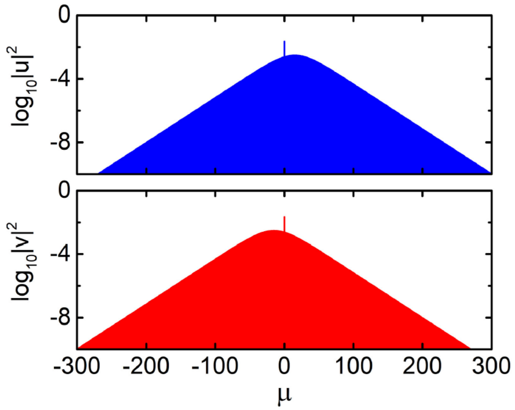
Relations, α: detuning, β: second order dispersion, γ: repetition difference, F: input

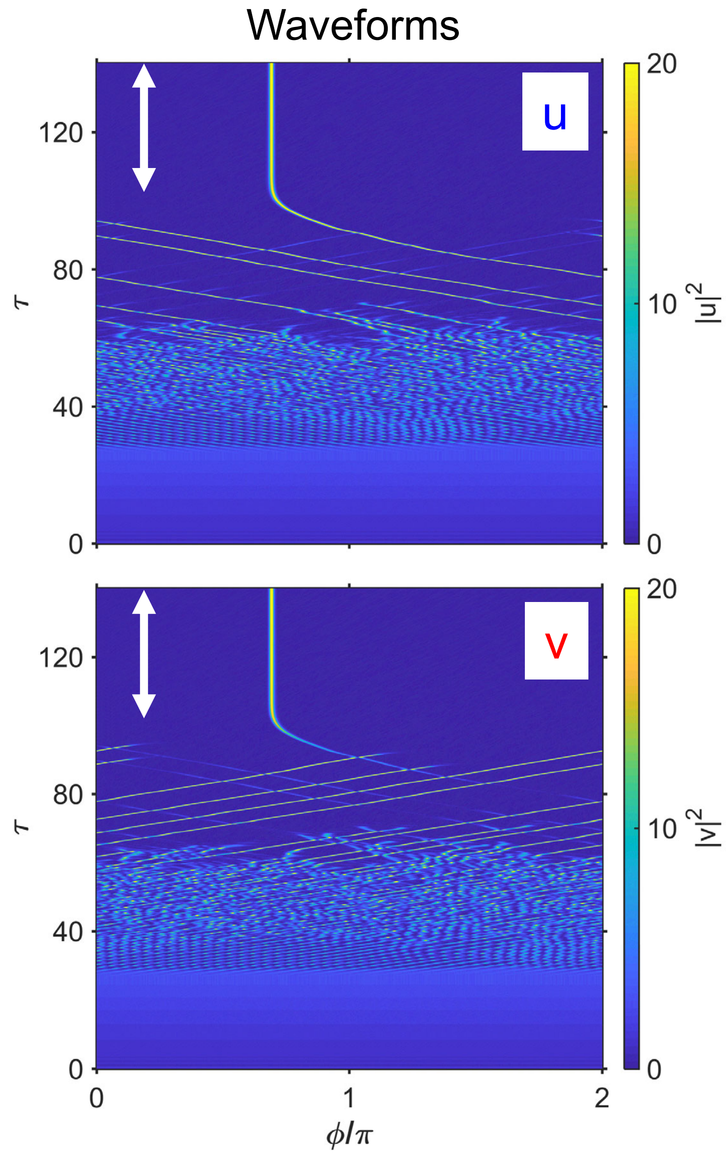
Trapping conditions as functions of F and δ

Group velocities are compensated with XPM

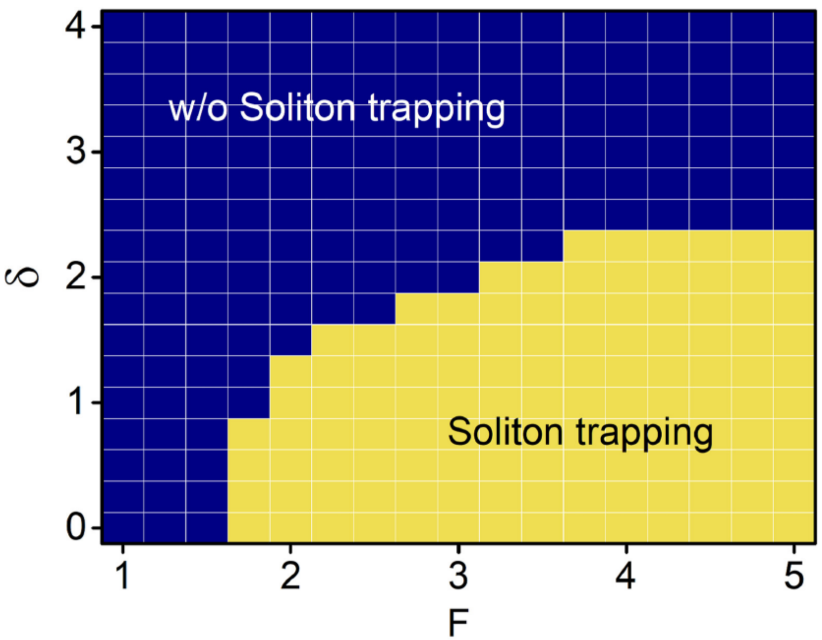
Center frequency shift:

$$\Delta\omega = \frac{\Delta D_1}{2D_2} \times D_1$$





Trapping conditions as functions of F and δ



Relations α: detuning, β: second order dispersion, γ: repetition difference, F: input, $\delta = \gamma(2\beta)^{-0.5}$

Analysis of coupled solitons solutions

Dimensionless coupled LLEs

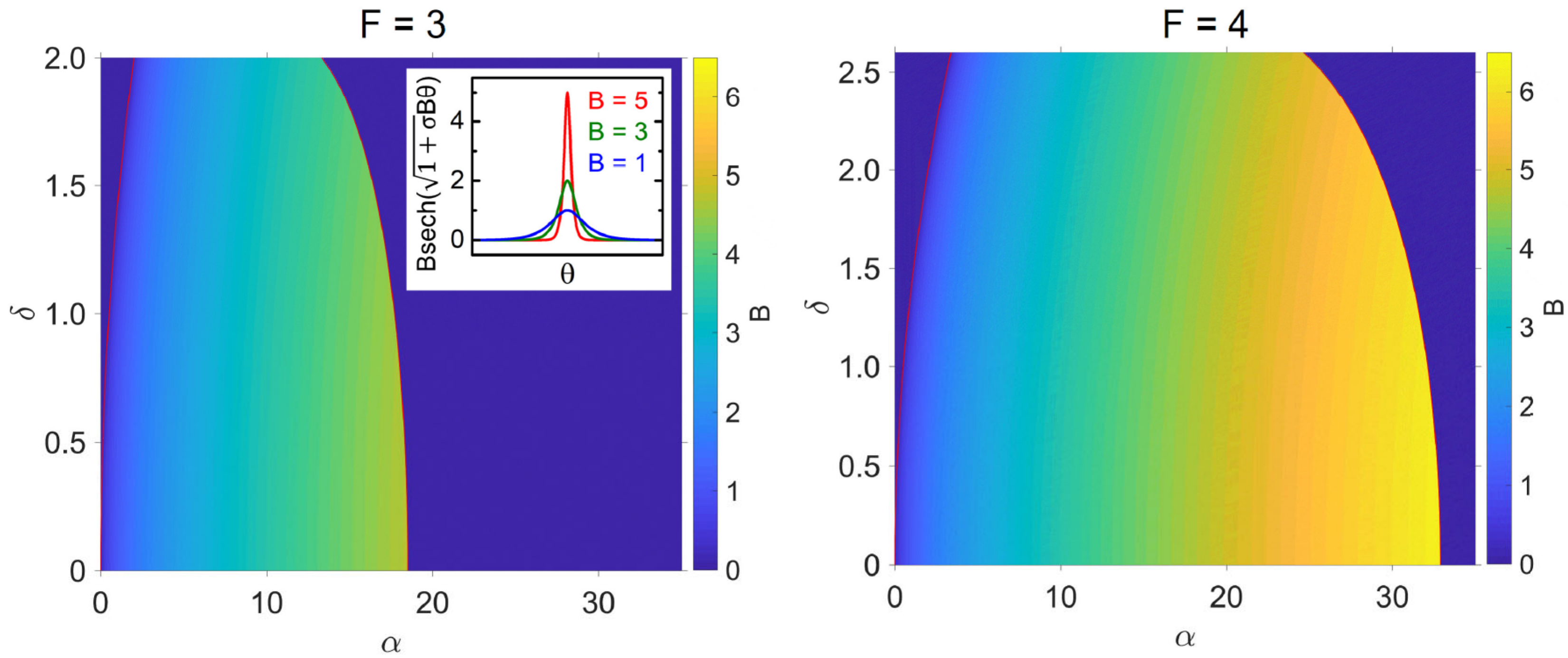
$$\theta = \frac{1}{\sqrt{2\beta}}\phi, \quad \delta = \frac{\gamma}{\sqrt{2\beta}}$$

$$\frac{\partial u}{\partial \tau} + i\alpha_{(u)}u - i\frac{1}{2}\frac{\partial^2 u}{\partial \theta^2} - i(|u|^2 + \sigma|v|^2)u - \delta\frac{\partial u}{\partial \theta} = F_{(u)} - u$$

$$\frac{\partial v}{\partial \tau} + i\alpha_{(v)}v - i\frac{1}{2}\frac{\partial^2 v}{\partial \theta^2} - i(|v|^2 + \sigma|u|^2)v + \delta\frac{\partial v}{\partial \theta} = F_{(v)} - v$$

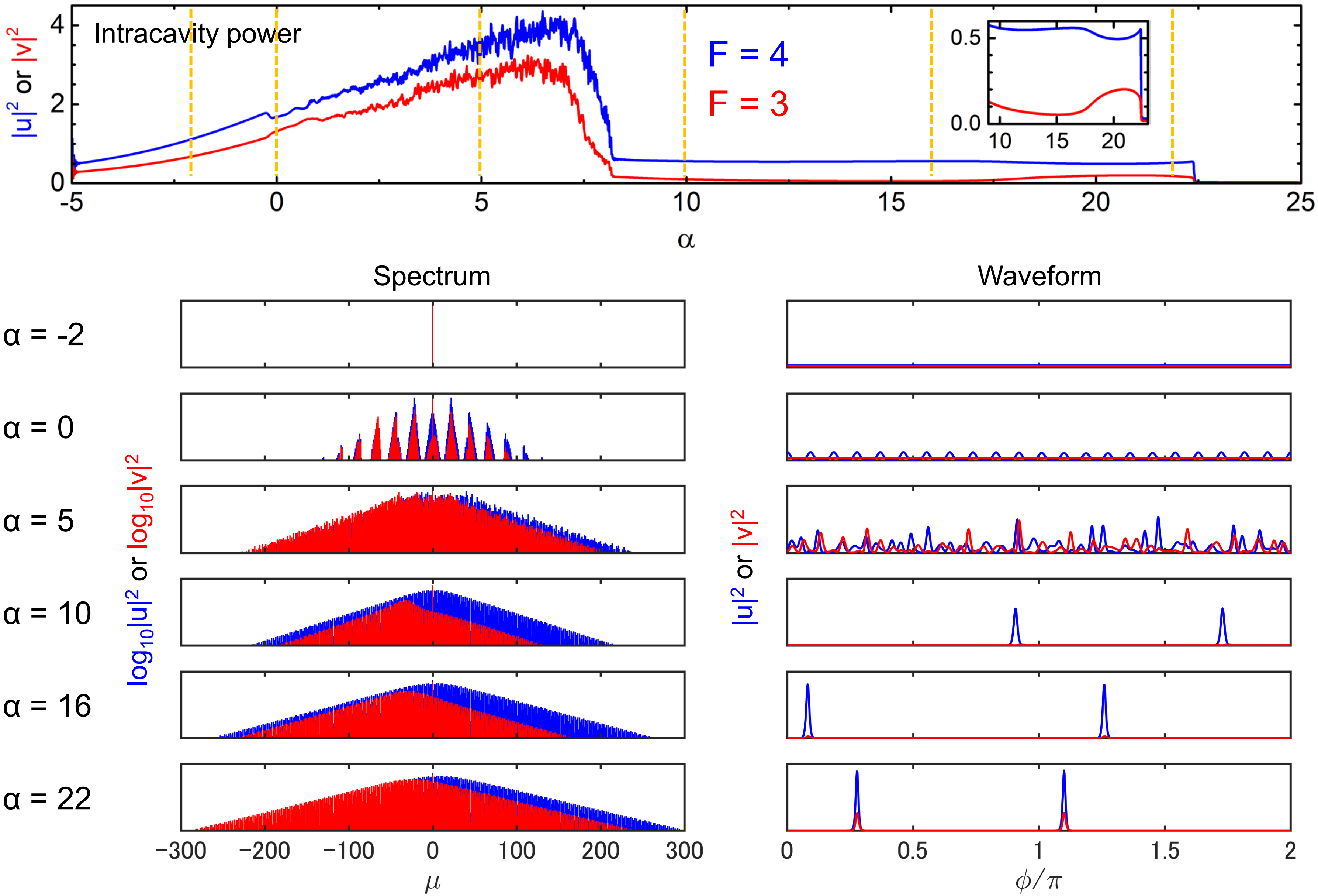
Ansatz of coupled solitons for perturbed Lagrangian approach

$$u = B\mathrm{sech}\left(\sqrt{1+\sigma B}\theta\right)\exp(i\varphi_0)\exp(i\delta\theta), \quad v = B\mathrm{sech}\left(\sqrt{1+\sigma B}\theta\right)\exp(i\varphi_0)\exp(-i\delta\theta)$$

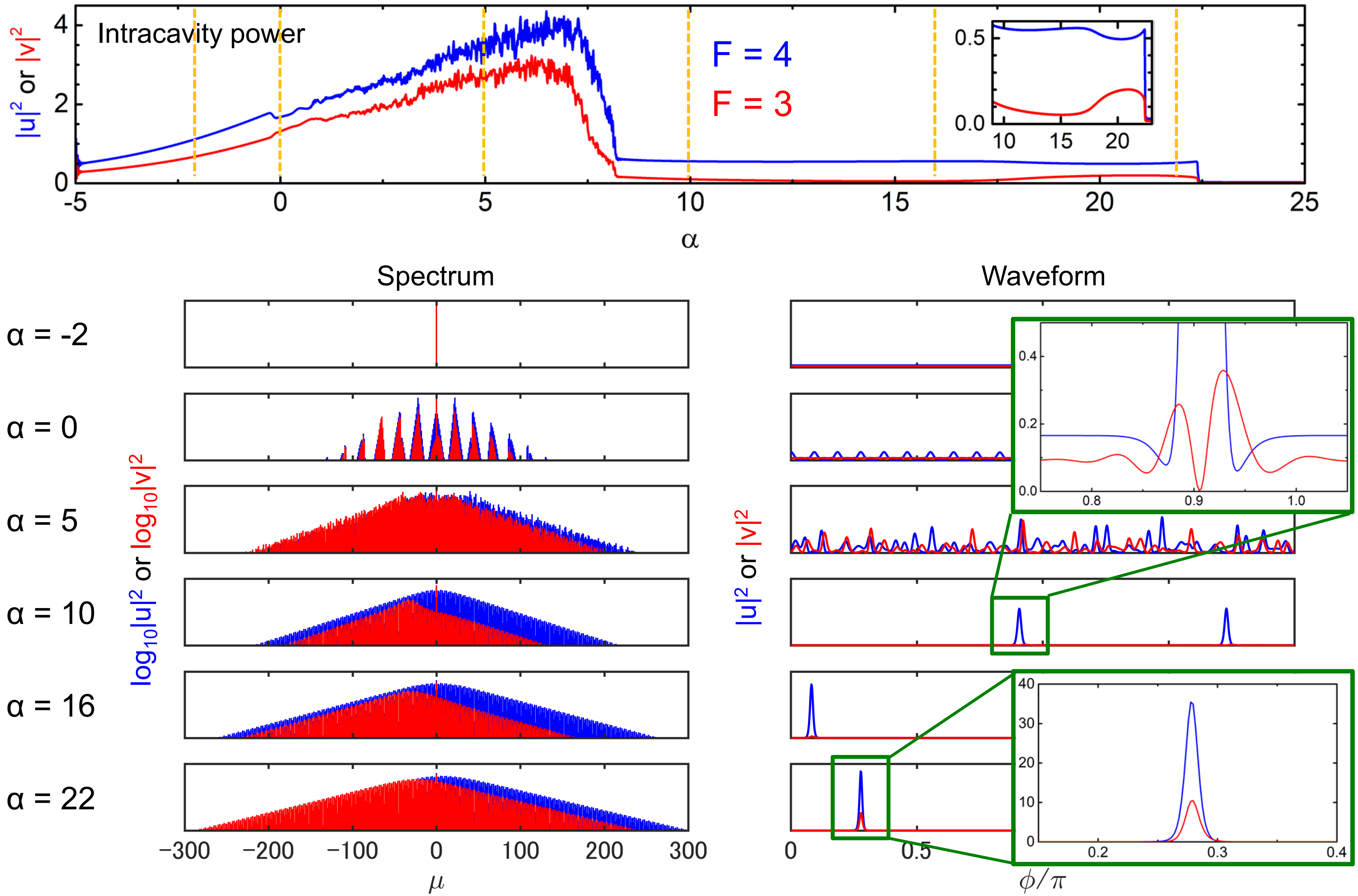


Relations, α: detuning, β: second order dispersion, γ: repetition difference, F: input, $\delta = \gamma(2\beta)^{-0.5}$

Strong soliton supports weak soliton generation



Strong soliton supports weak soliton generation



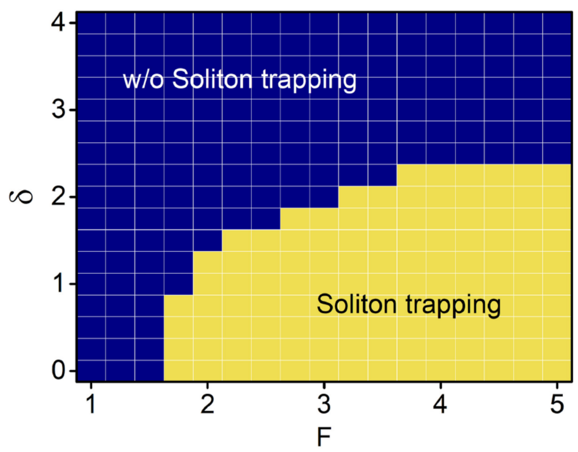
Summary

- Developed simulation model with coupled LLEs, which include XPM and repetition difference terms
- Calculated with generalized parameters to reveal trapping conditions
- Performed analysis of coupled solitons solutions

$$\frac{\partial u}{\partial \tau} = -\left(1 + i\alpha_{(u)}\right)u + i\frac{1}{2}\frac{\partial^2 u}{\partial \theta^2} + i(|u|^2 + \sigma|v|^2)u + F_{(u)} + \delta\frac{\partial u}{\partial \theta}$$

$$\frac{\partial v}{\partial \tau} = -\left(1 + i\alpha_{(v)}\right)v + i\frac{1}{2}\frac{\partial^2 v}{\partial \theta^2} + i(|v|^2 + \sigma|u|^2)v + F_{(v)} - \delta\frac{\partial v}{\partial \theta}$$

Trapping conditions



Analysis of coupled solitons solutions