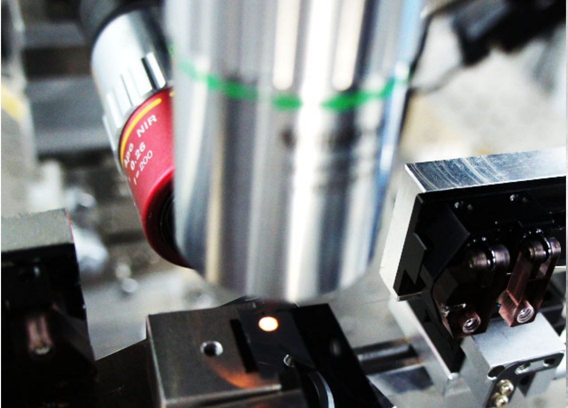
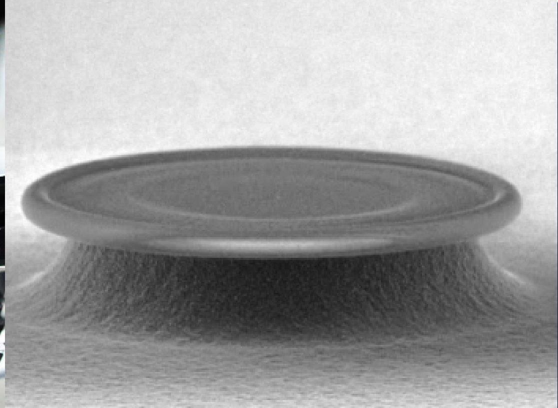
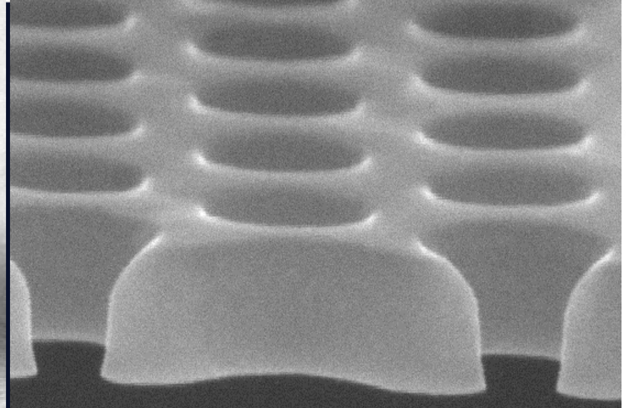
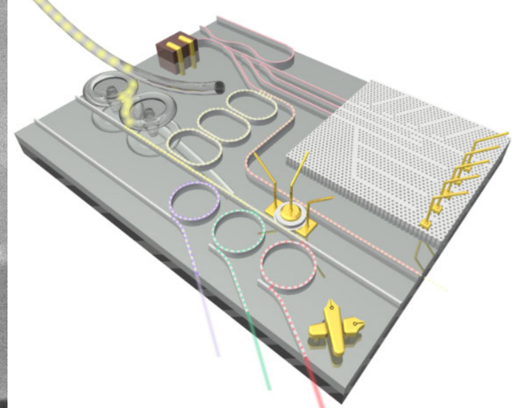





Microresonators and Solitons II

SPIE Photonics West

Kerr comb generation in a mode coupled system

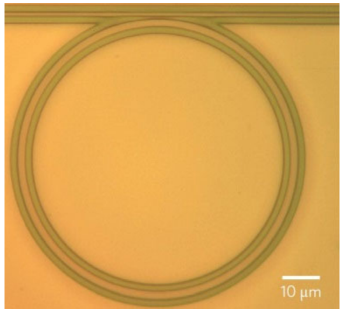
Takasumi Tanabe, Shun Fujii, and Ruo Suzuki

Electronics and Electrical Engineering, Keio University, Japan

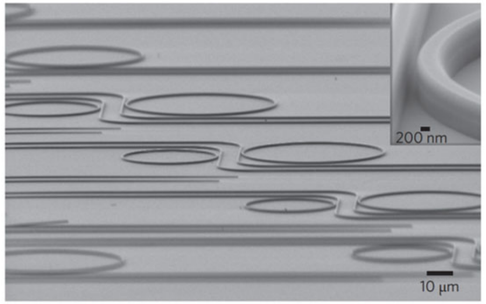
takasumi@elec.keio.ac.jp

Tuesday 5 February, 2019 1:50PM – 2:15PM
Room 303 (South Level Three)

High-Q whispering-gallery mode microcavities

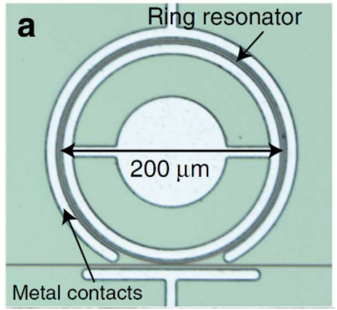


Silicon nitride
Weiner group (Purdue)

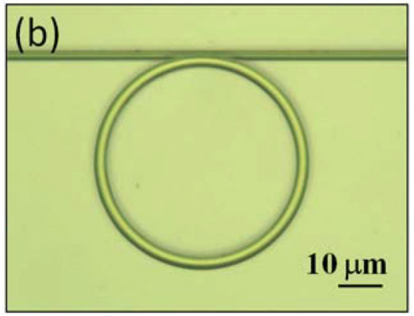


Diamond
Loncar group (Harvard)

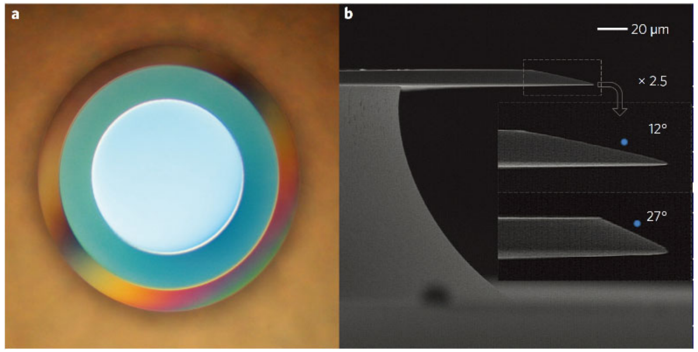
Crystalline (CaF_2, MgF_2, etc)
Kippenberg group (EPFL, Swiss),
Makei group (OE Waves)



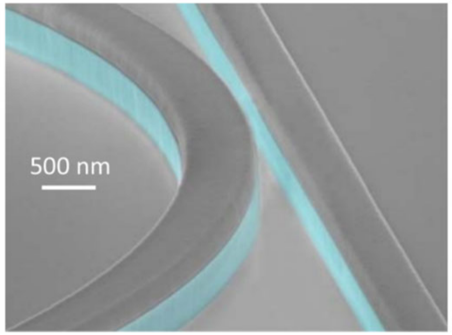
Silicon
Gaeta group (Columbia)



AlN
Tang group (Yale)



Silica
Vahala group (Caltech)



AlGaAs
Yvind group (DTU, Denmark)

◆Q-factor

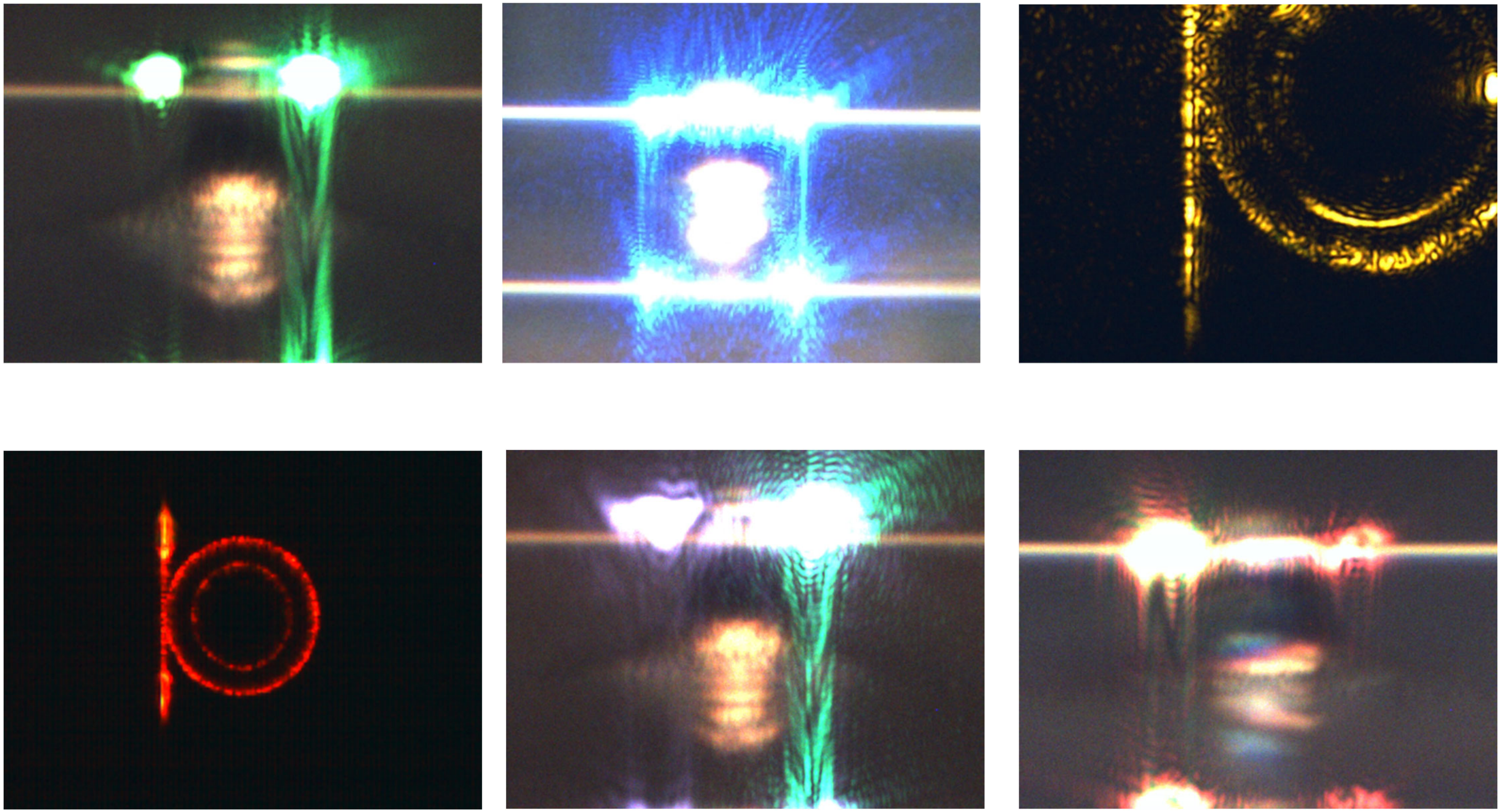
$$Q = \omega \times \frac{\text{stored energy}}{\text{power in/out}}$$

◆Photon density

$$\propto Q/V$$

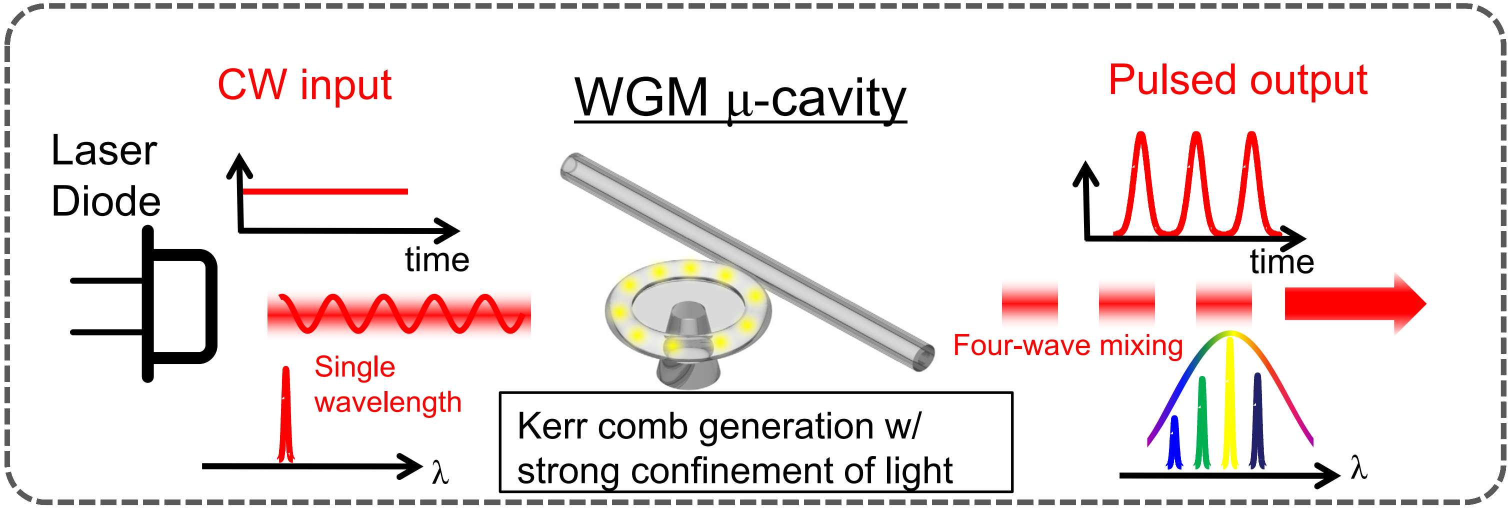
Third-harmonic generations in toroid microcavity

S. Fujii, *et al.*, Opt. Lett. **42**, 2010 (2017).
A. C.-Jinnai, *et al.*, Opt. Express **24**, 26322 (2016).

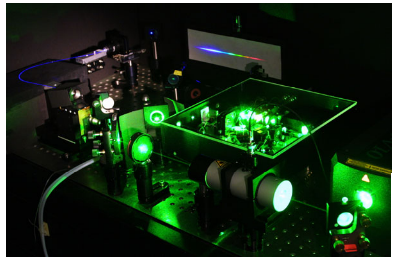


Kerr comb in microcavity system

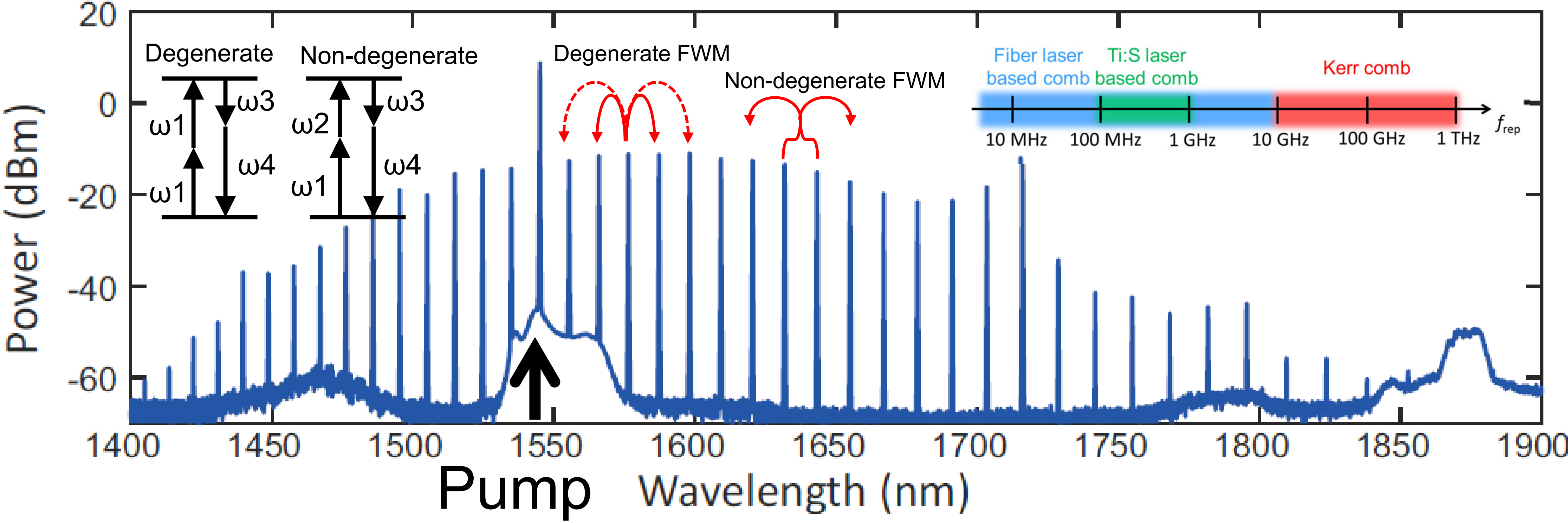
Convert CW laser to ultrashort pulse train w/ > 800 GHz rep. rate



Ti:Sapphire laser based comb

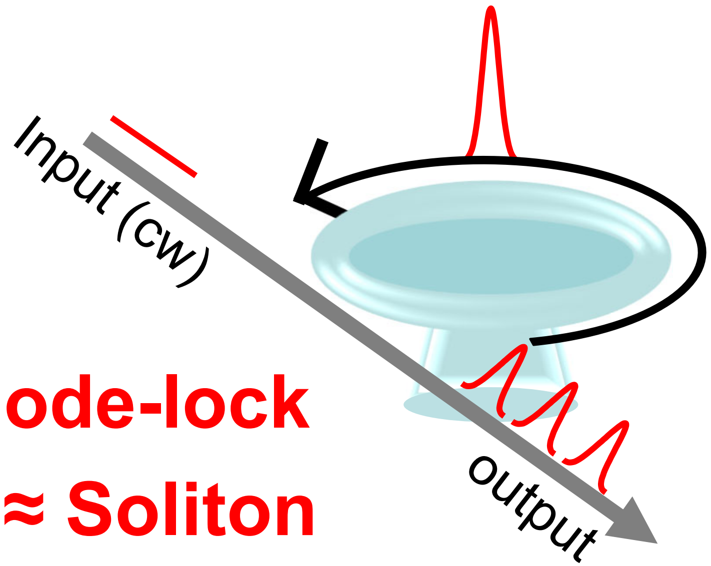


large & expensive



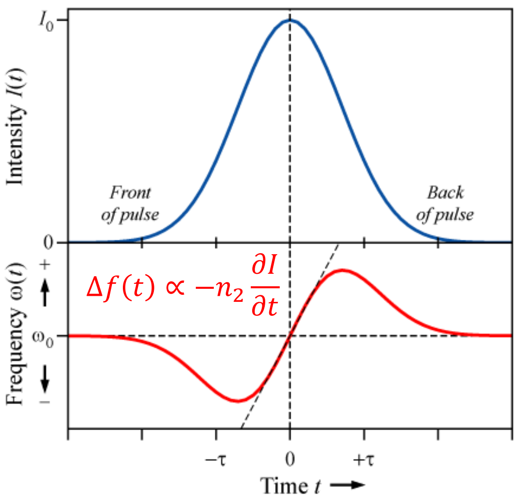
Required conditions for soliton formation

▶ Soliton in a μ-cavity



Mode-lock

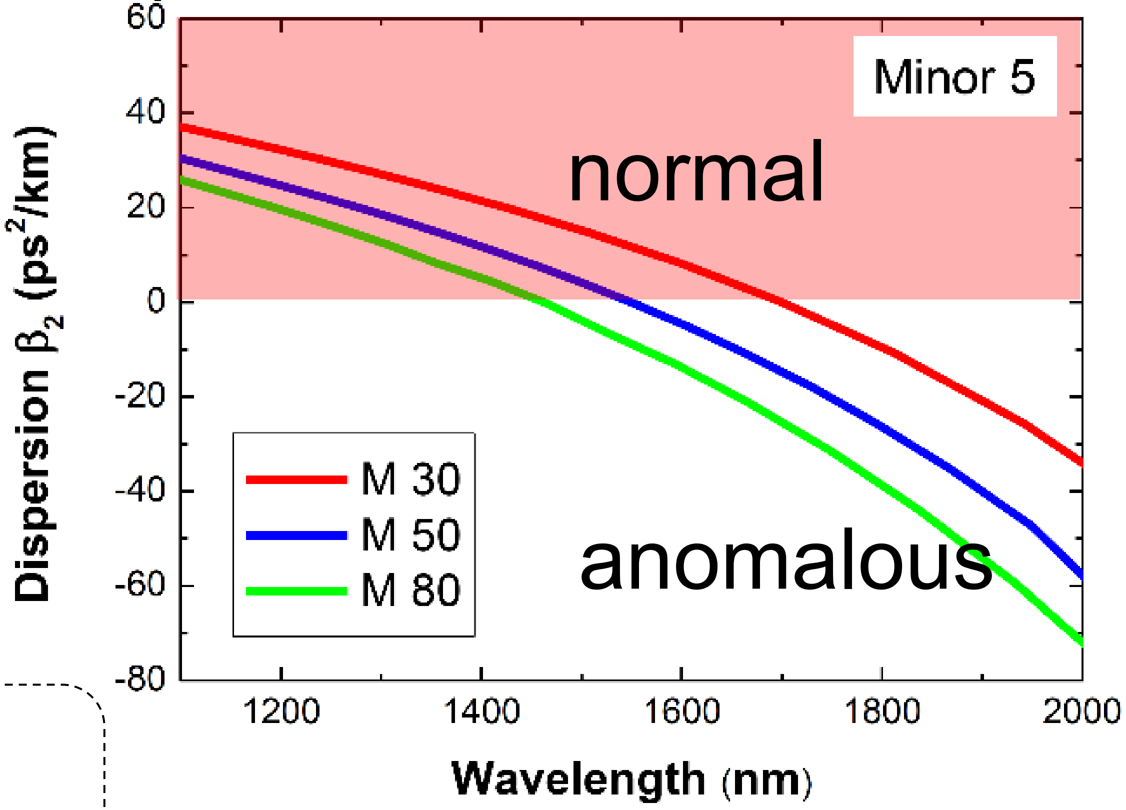
≈ Soliton



Nonlinear (Kerr) phase

≈ Anomalous dispersion

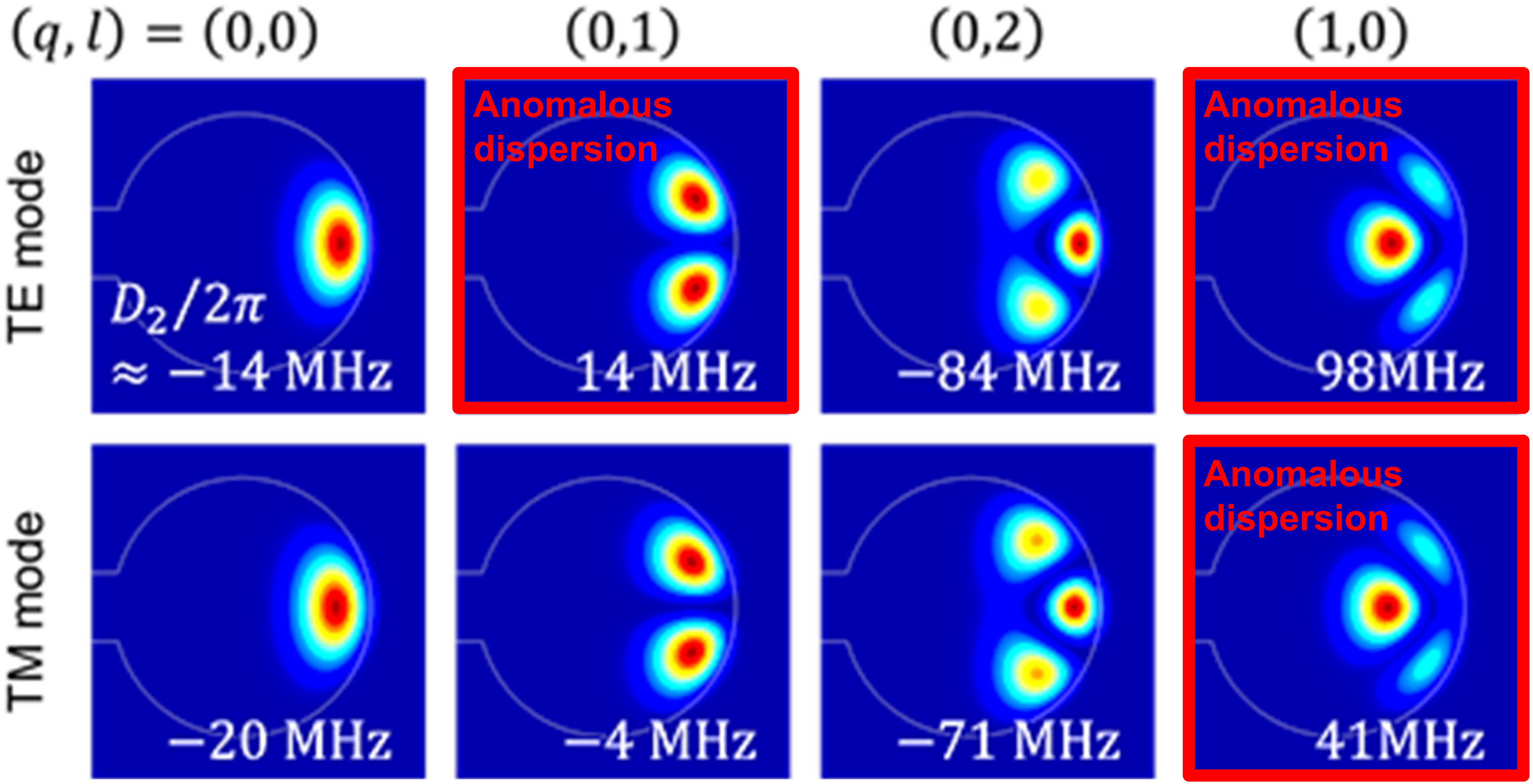
▶ Dispersion in a small toroid



- Material dispersion
- Geometric dispersion

S. Fujii, *et al.*, Opt. Lett. **42**, 2010 (2017).

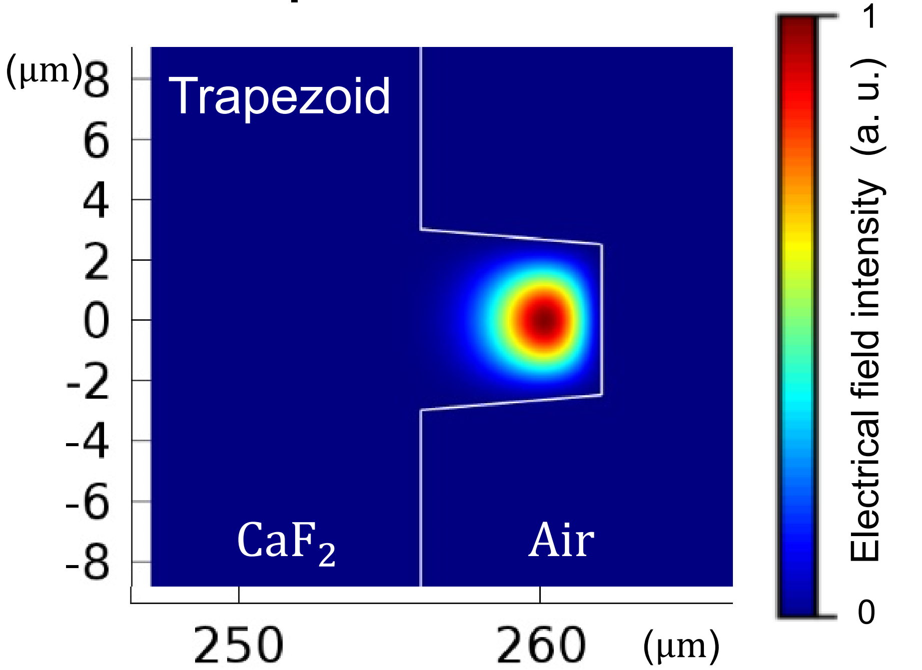
Dispersions in toroid microcavity (r = 35 μm)



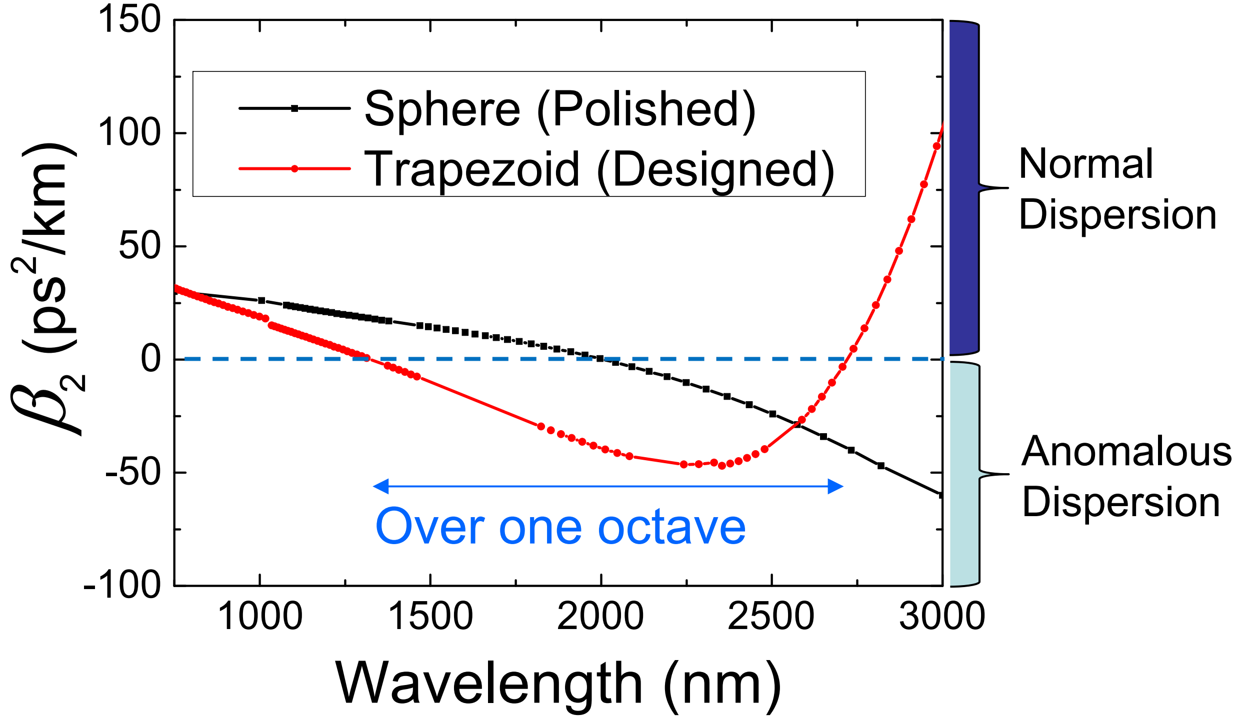
Design the dispersion

Y. Nakagawa, *et al.*, J. Opt. Soc. Amer. B **33**, 1913 (2016).

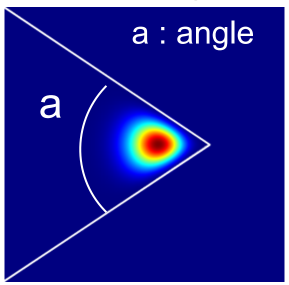
▶Mode profile



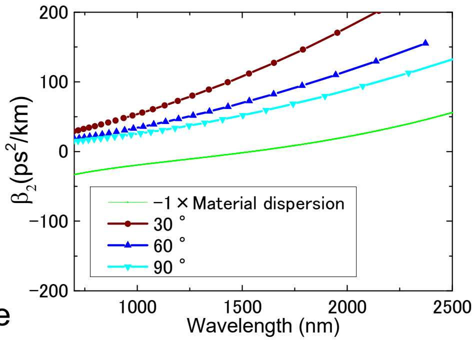
▶Dispersion (geometric + material)



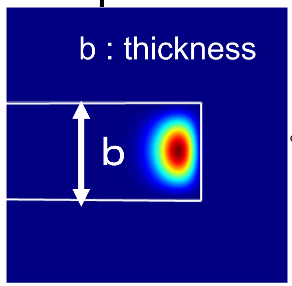
Triangular



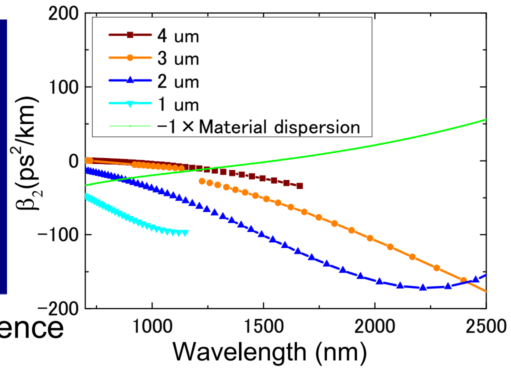
Angle dependence



Square



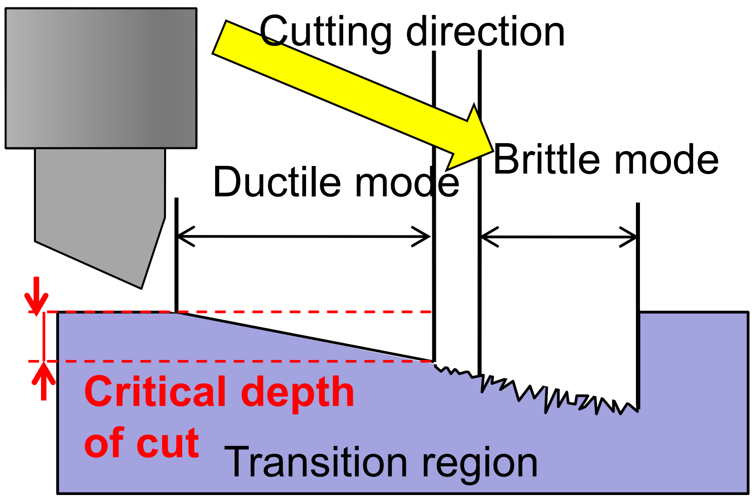
Thickness dependence

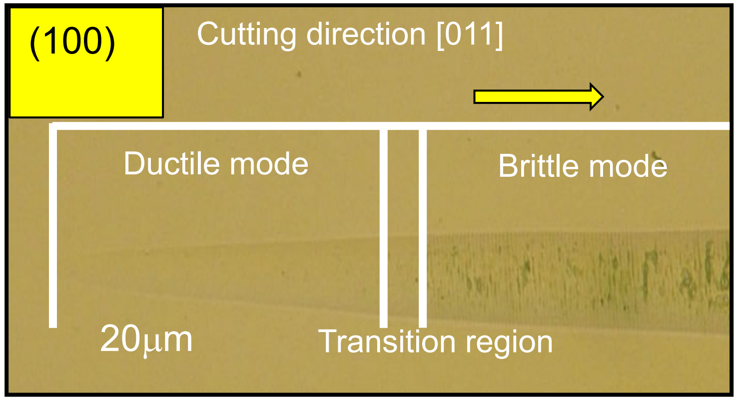


Fabrication of CaF_2 WGM cavity w/ cutting

Precise machining process

Y. Mizumoto, *et al*., Procedia Eng. **19,** 264 (2011).
S. Azami, *et al*. Procedica CIRP **13,** 225 (2014).

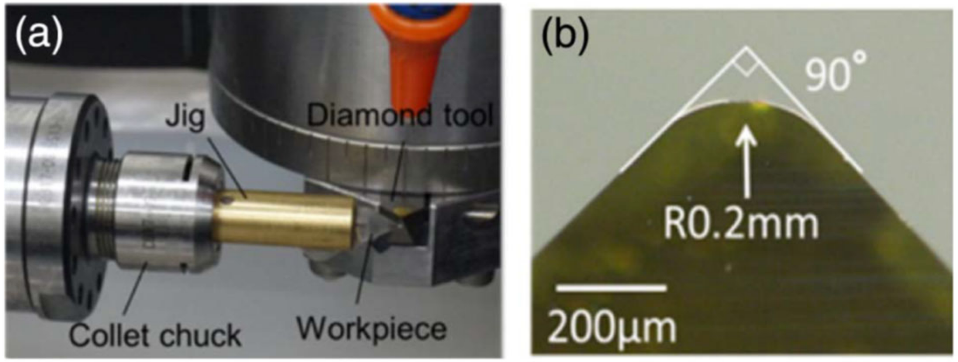


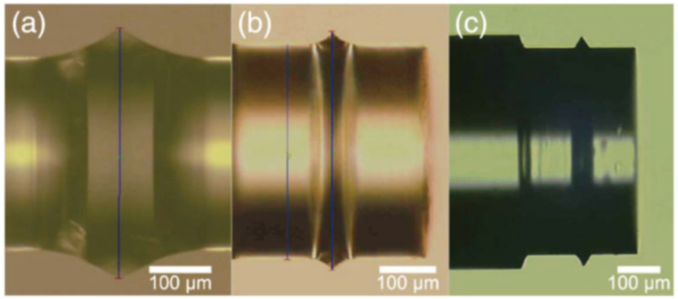


➢ CaF_2 can be smoothly cut in ductile mode cutting

Computer controlled lathe cutting

Y. Nakagawa, *et al*., J. Opt. Soc. Amer. B **33**, 1913 (2016).

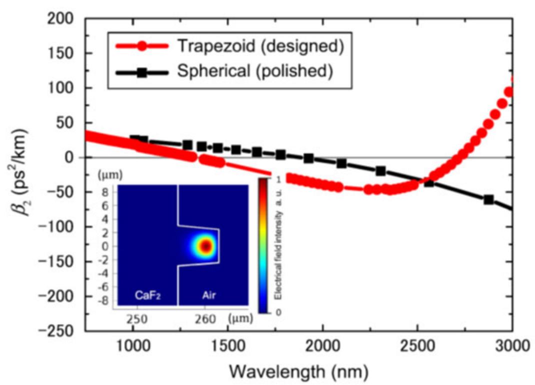


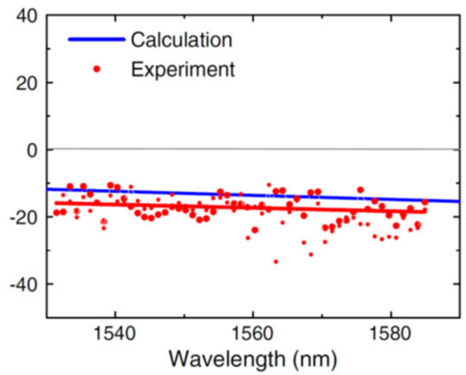


$R_{\rm rms} = 3$ nm

$\boldsymbol{Q = 3.0 \times 10^7}$

Preliminarily (2018/Nov.)

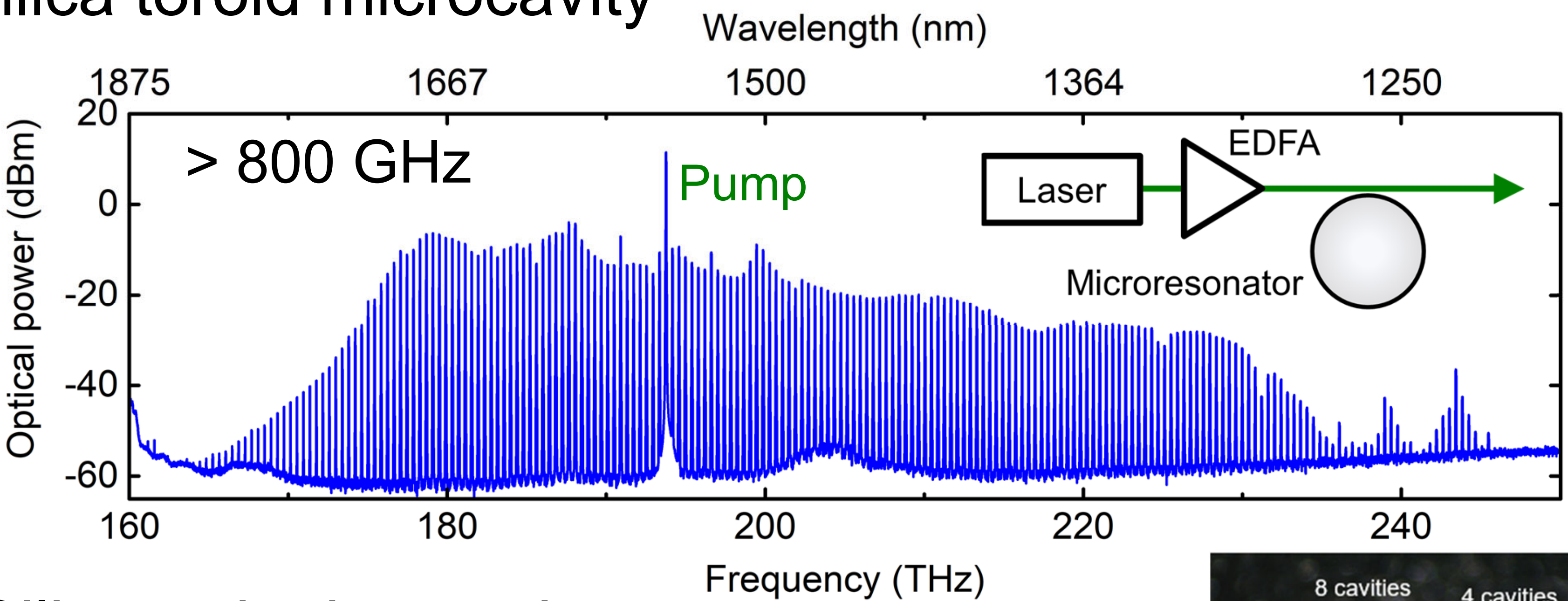


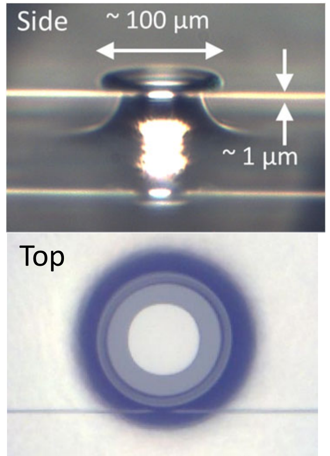


Kerr comb in microcavity system

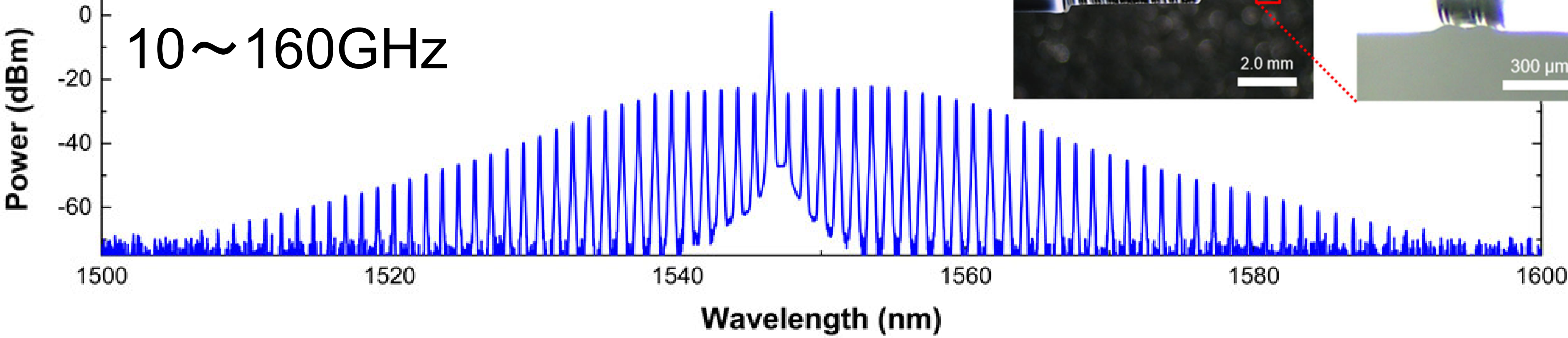
R. Suzuki, *et al.*, J. Opt. Soc. Amer. B **35**, 933 (2018). (**Editor's pick**)
S. Fujii, *et al.,* J. Opt. Soc. Amer. B **35**, 100 (2018). (**Editor's pick**)

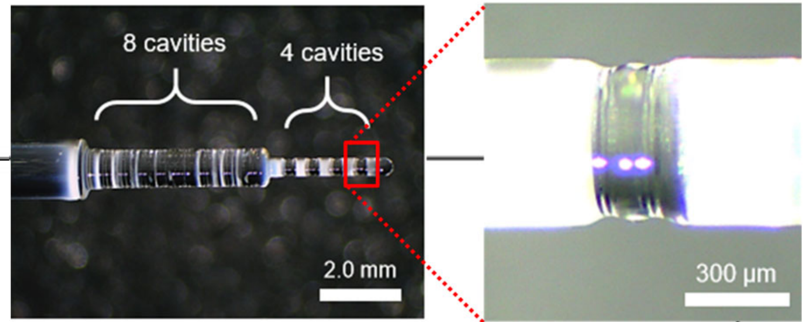
▶Silica toroid microcavity



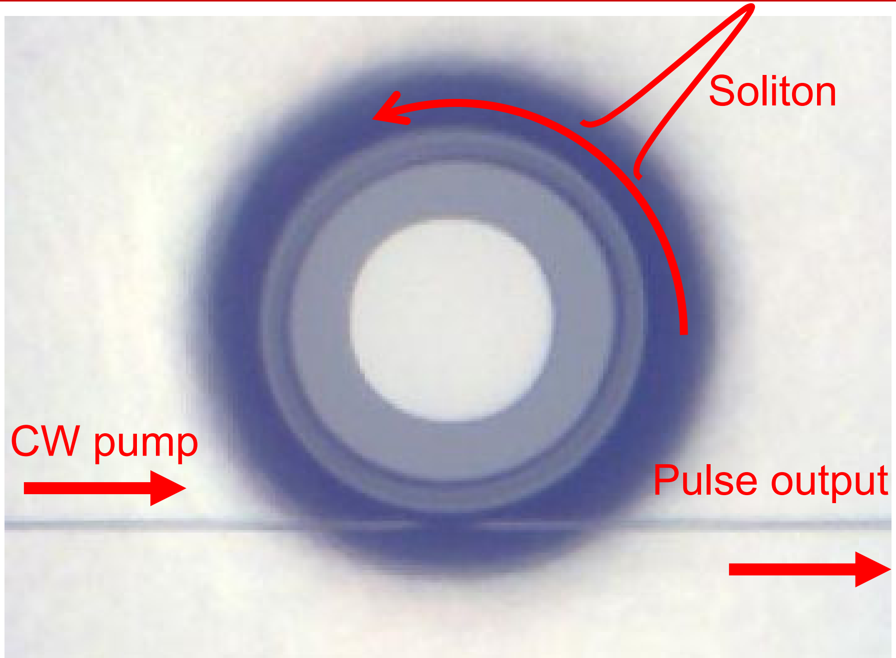


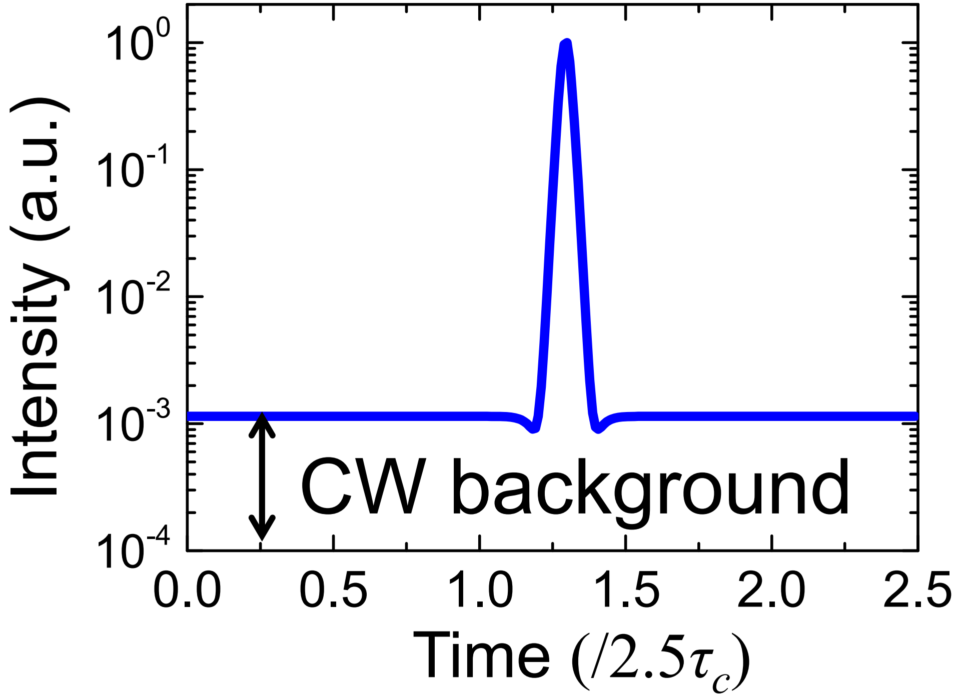
▶Silica rod microcavity



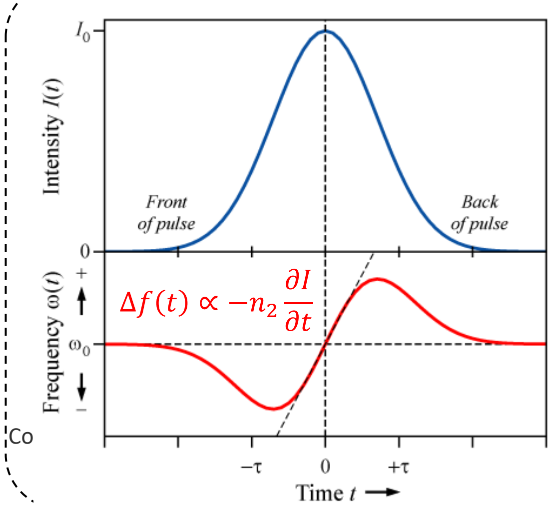


Efficiency and the use of dark pulse (normal dispersion)





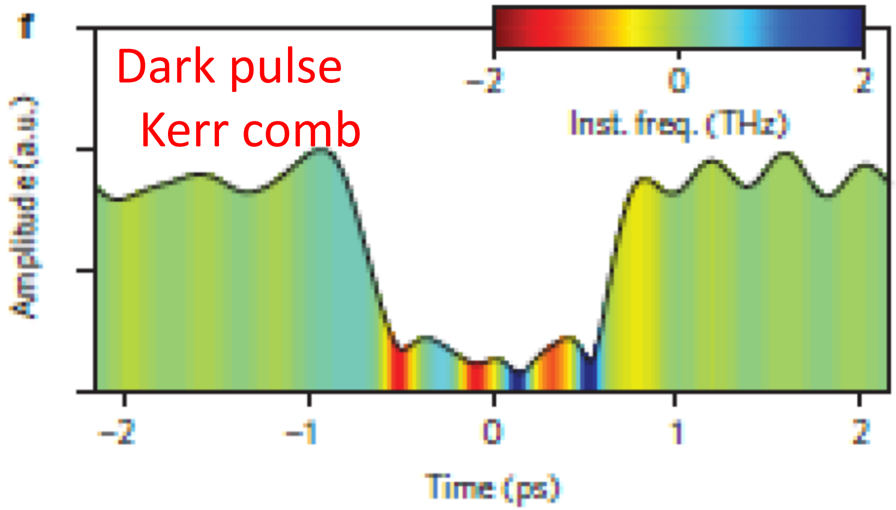
⇨ Coupling is low w/ a bright pulse



Nonlinear (Kerr) phase

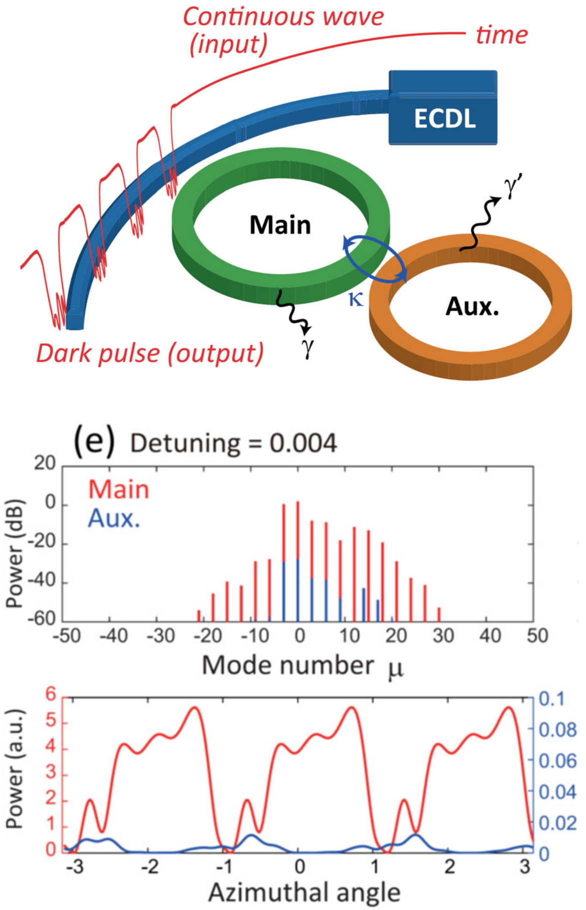
≈ Normal dispersion

X. Xue et al., Nat. Photon. **10** (2015)



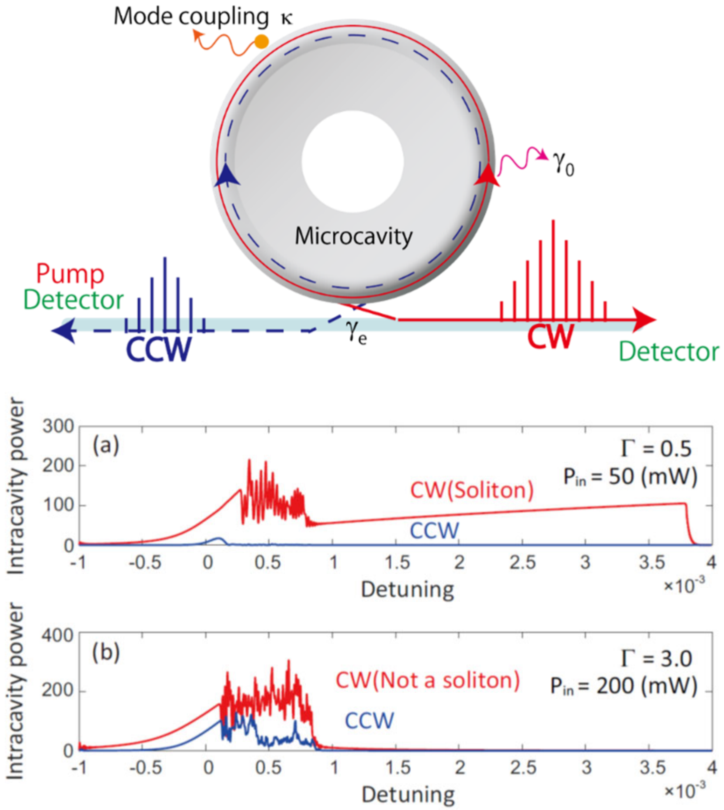
Kerr combs with mode coupling

Two modes coupling



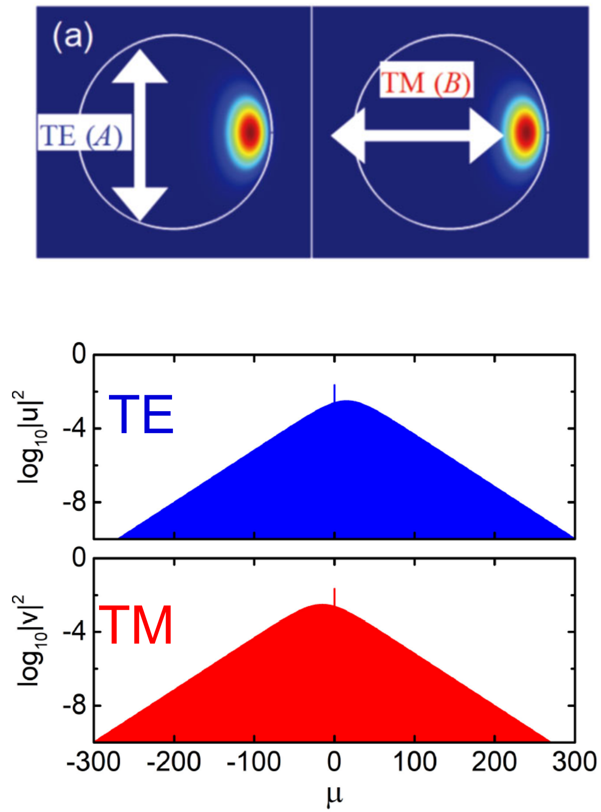
S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).

CW/CCW mode coupling



S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

TE/TM mode coupling



R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

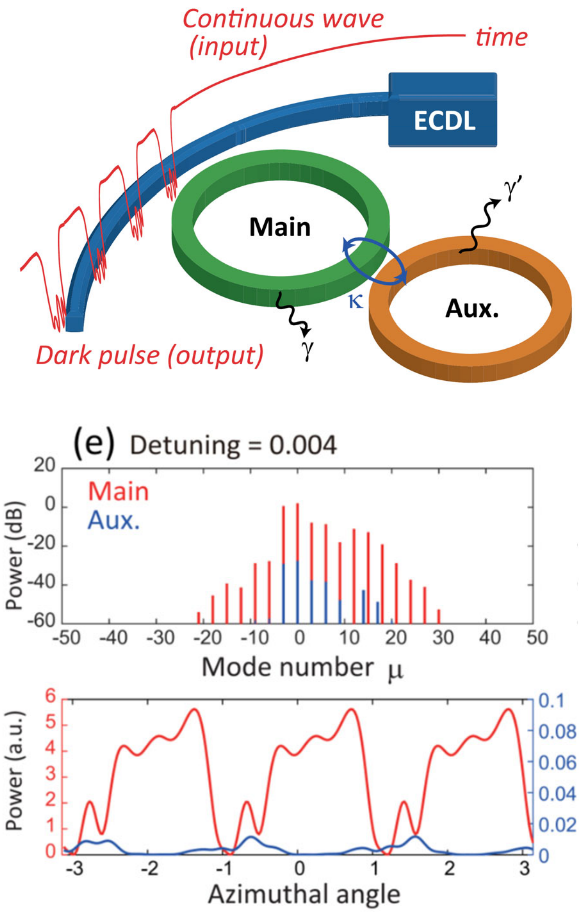
Dark pulse generation at normal dispersion

Effect of inherent coupling

Dual comb generation

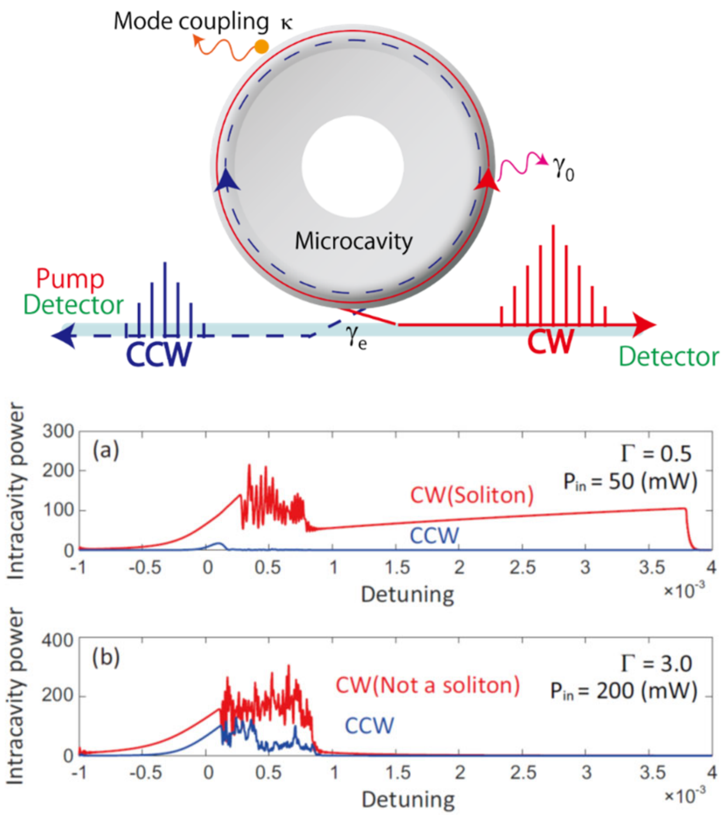
Kerr combs with mode coupling

Two modes coupling



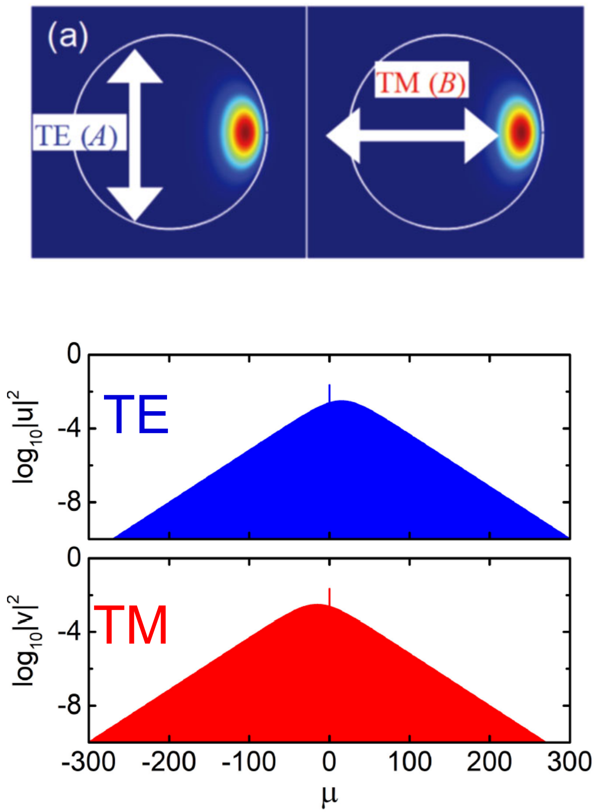
S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).

CW/CCW mode coupling



S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

TE/TM mode coupling



R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

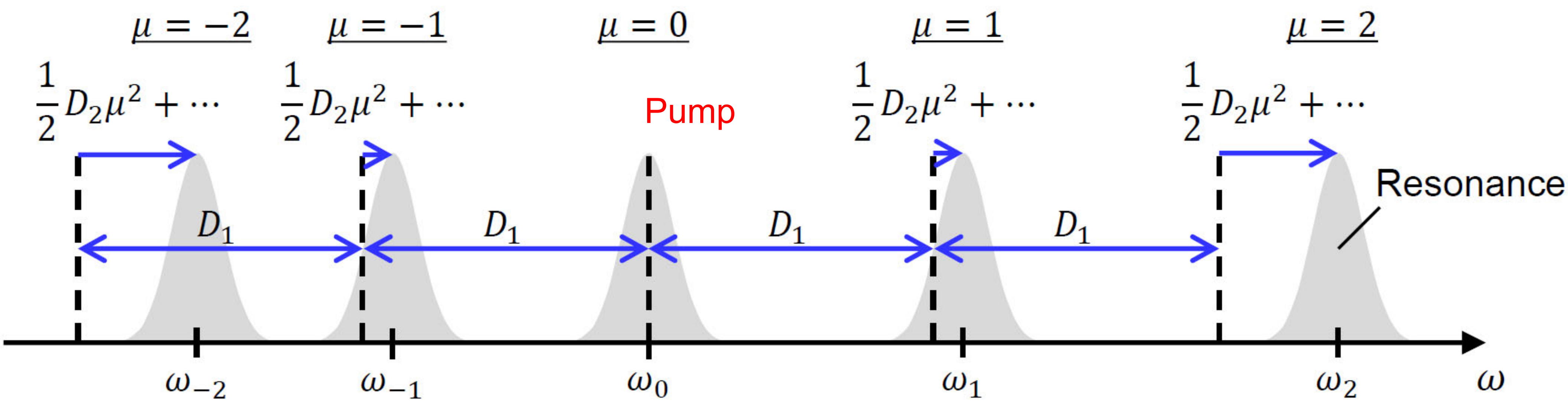
Dark pulse generation at normal dispersion

Effect of inherent coupling

Dual comb generation

Dispersion in a cavity (spectrum domain picture)

S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).



Resonance frequencies (μ is mode number)

$$\omega_\mu = \omega_0 + D_1\mu + \frac{1}{2}D_2\mu^2 + \cdots$$

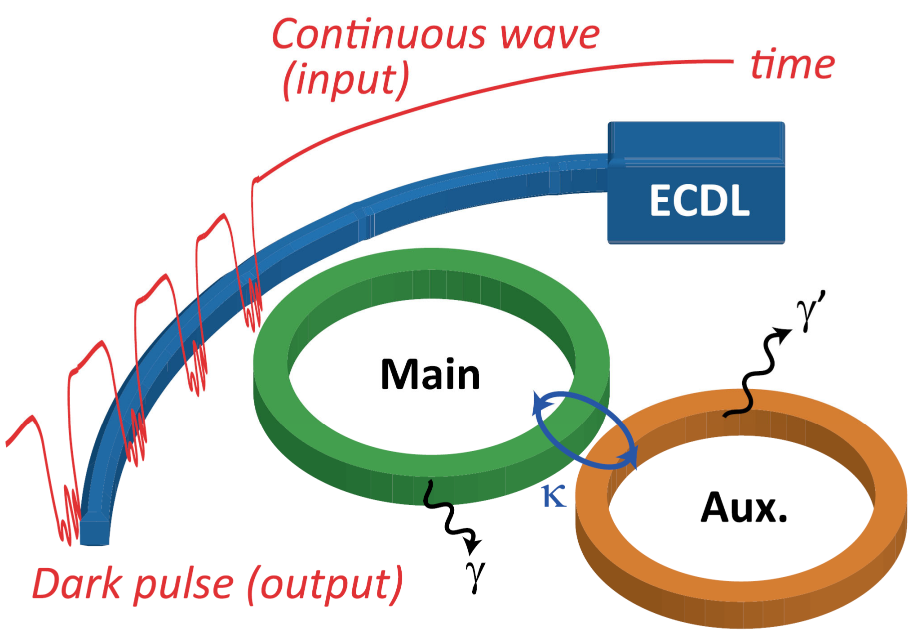
$D_2 > 0$: Anomalous dispersion

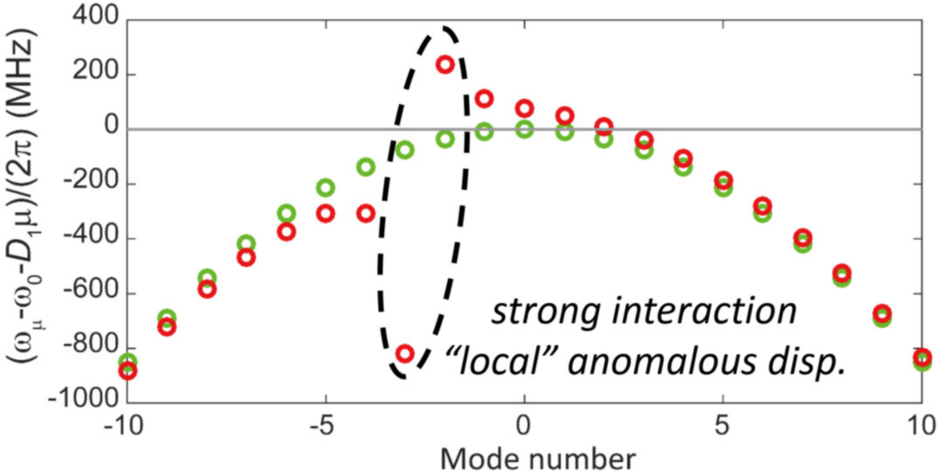
$D_2 < 0$: Normal dispersion

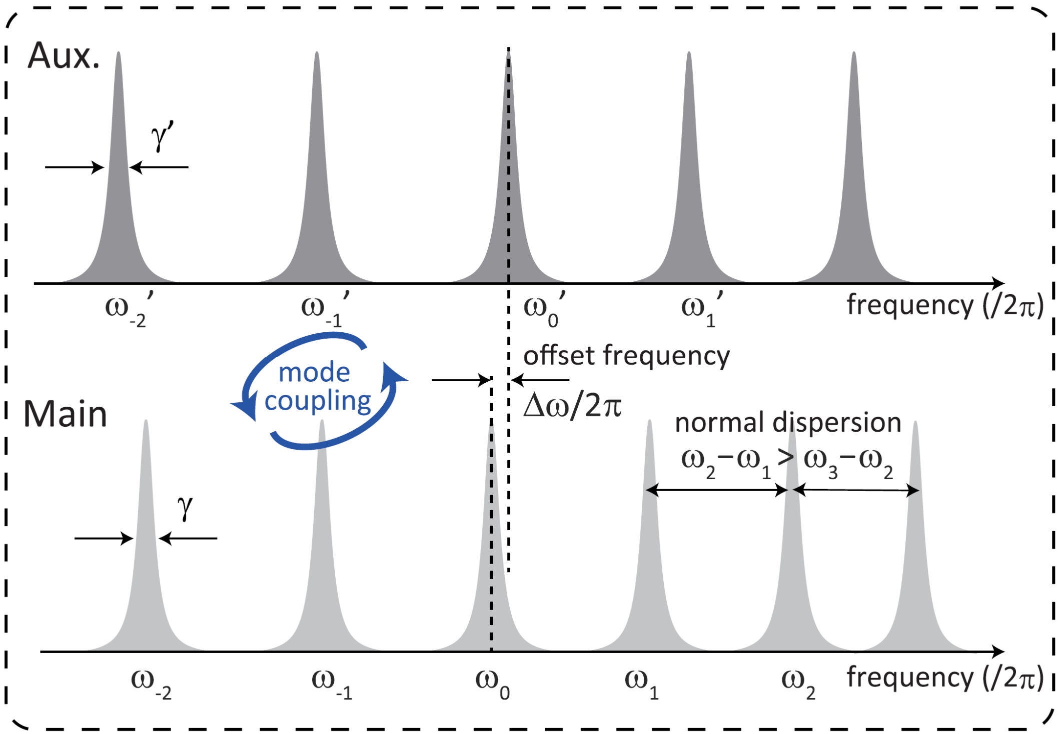
Always need anomalous dispersion to have modulation instability gain

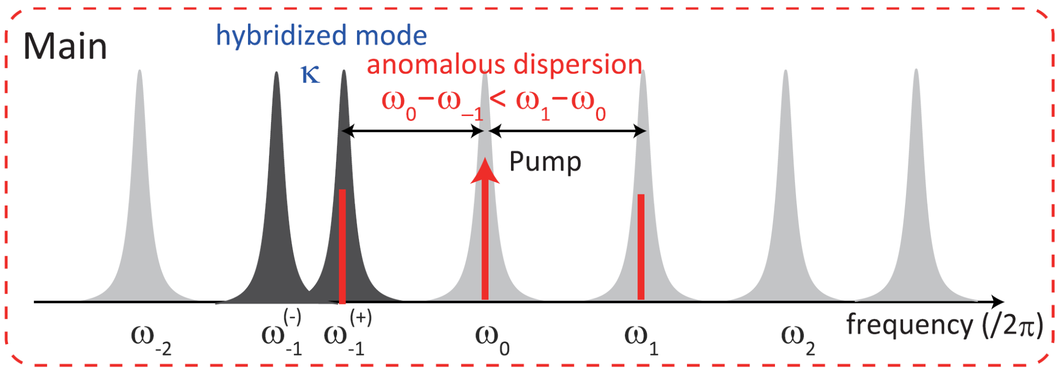
Nonlinear Coupled Mode Equations for Kerr Comb Generation in Coupled Microcavity System

S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).





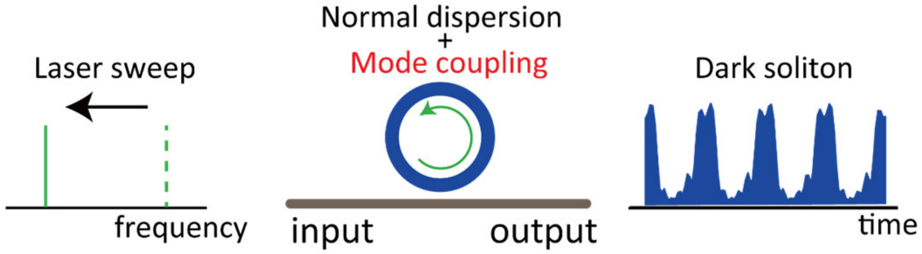


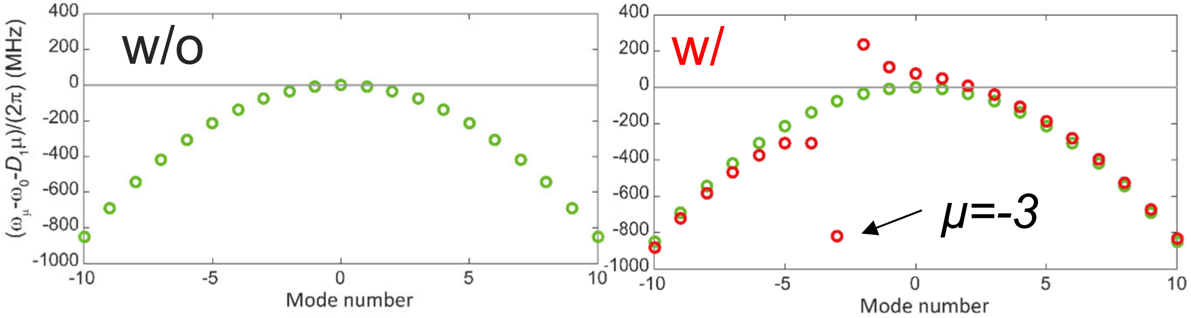


Mode coupling is the key

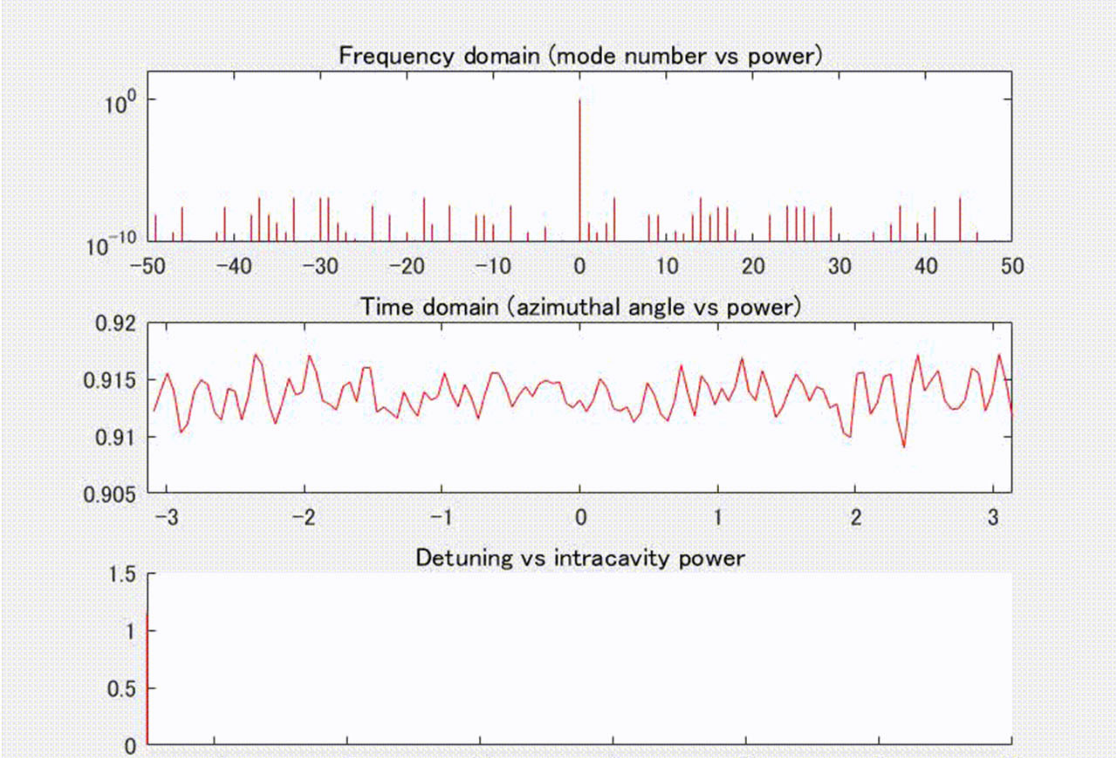
Dark pulse generation w/ mode coupling

S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).



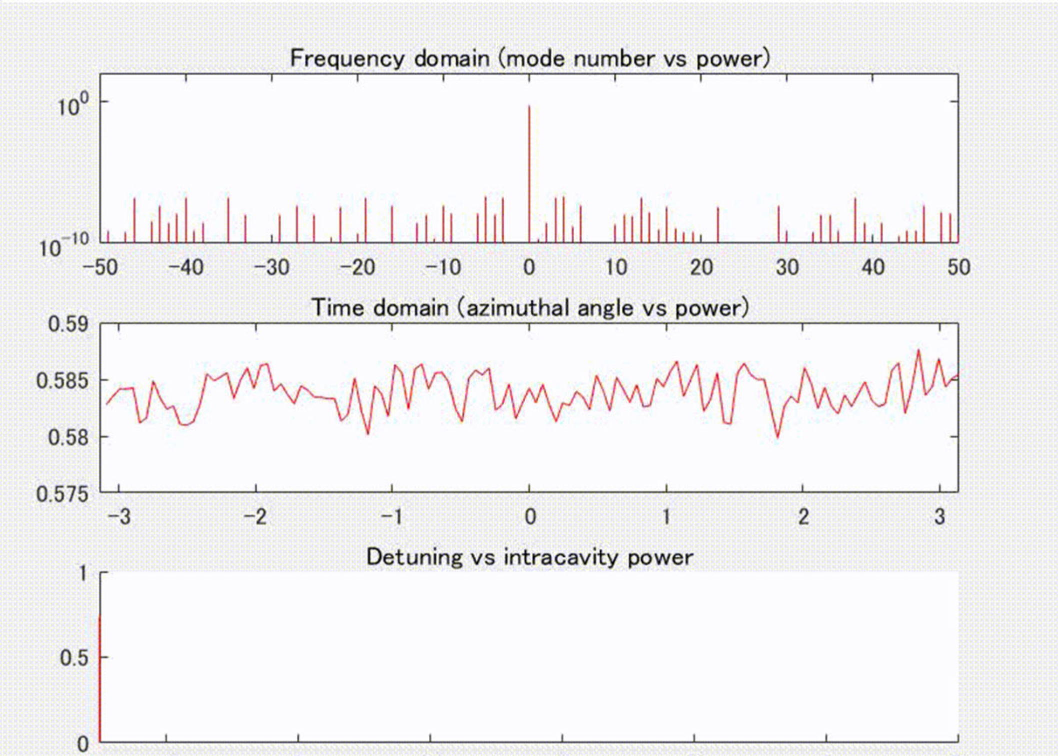


w/o mode coupling κ = 0 (rad GHz)



✓ ***No comb formation***

w/ mode coupling κ = 3.34 (rad GHz)

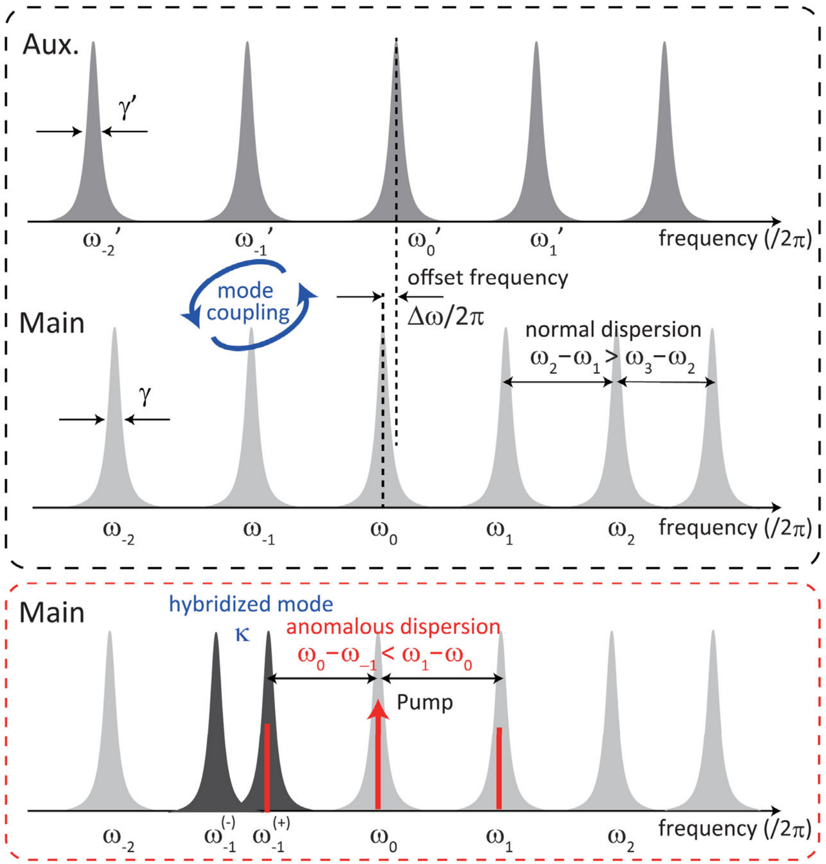


✓ ***3-FSR comb & dark pulse formation***

Mode coupling and MI gain

S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).

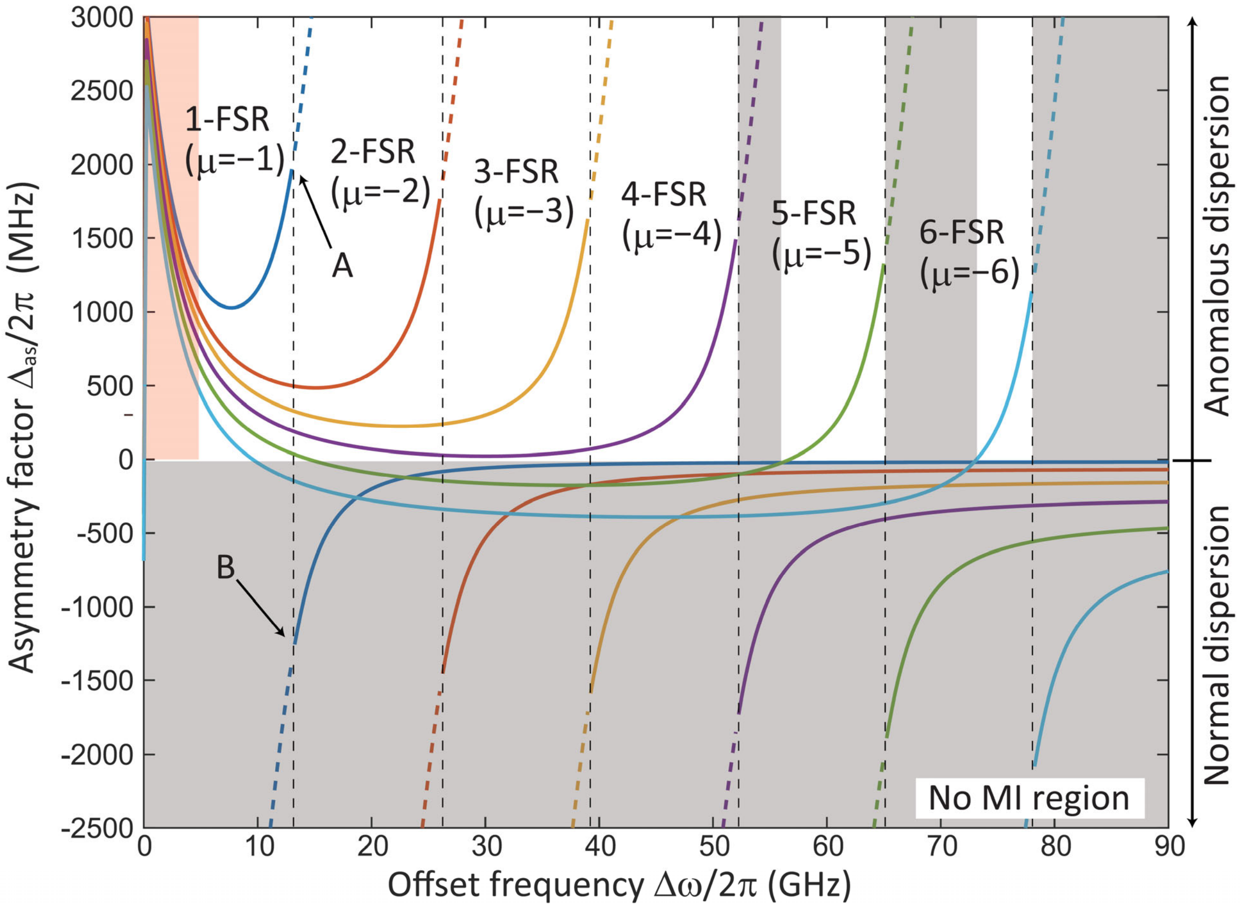
▶Mode coupling



$$\Delta_{as} = (\omega_\mu - \omega_0) - (\omega_0 - \omega_{-\mu})$$

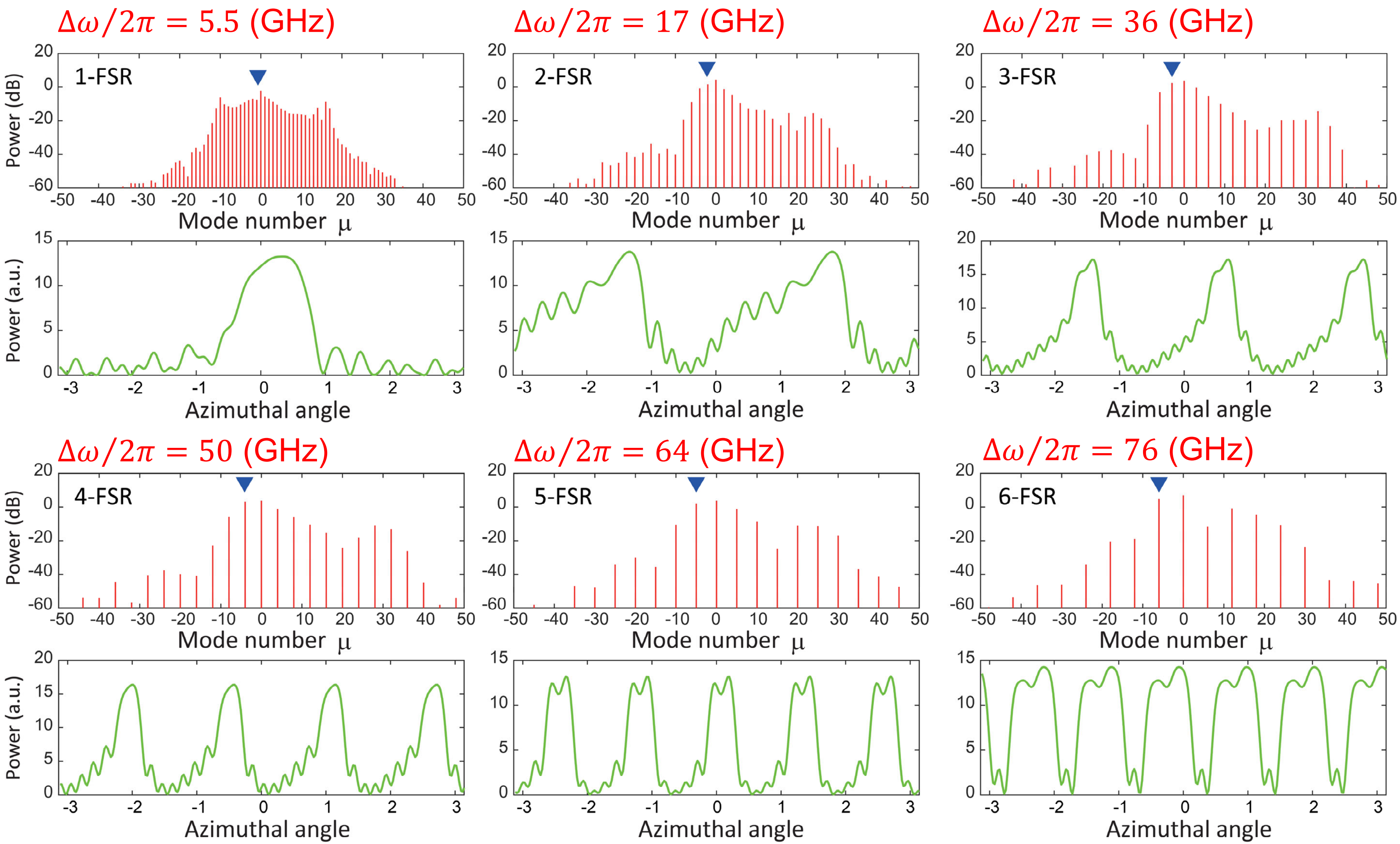
Δ_{as} > Anomalous dispersion
Δ_{as} < Normal dispersion

▶Phase matching (MI gain)



Deterministic FSR generation

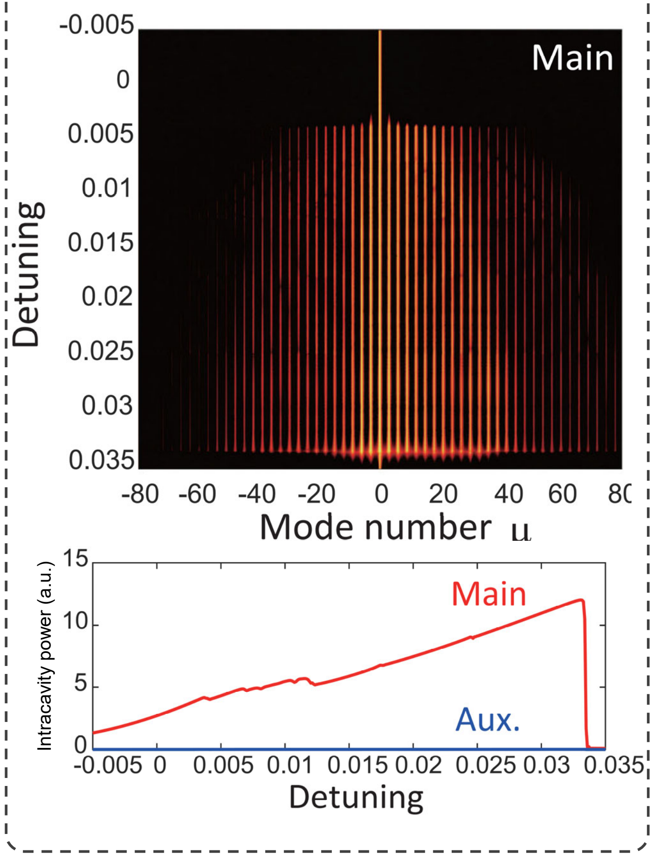
S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).



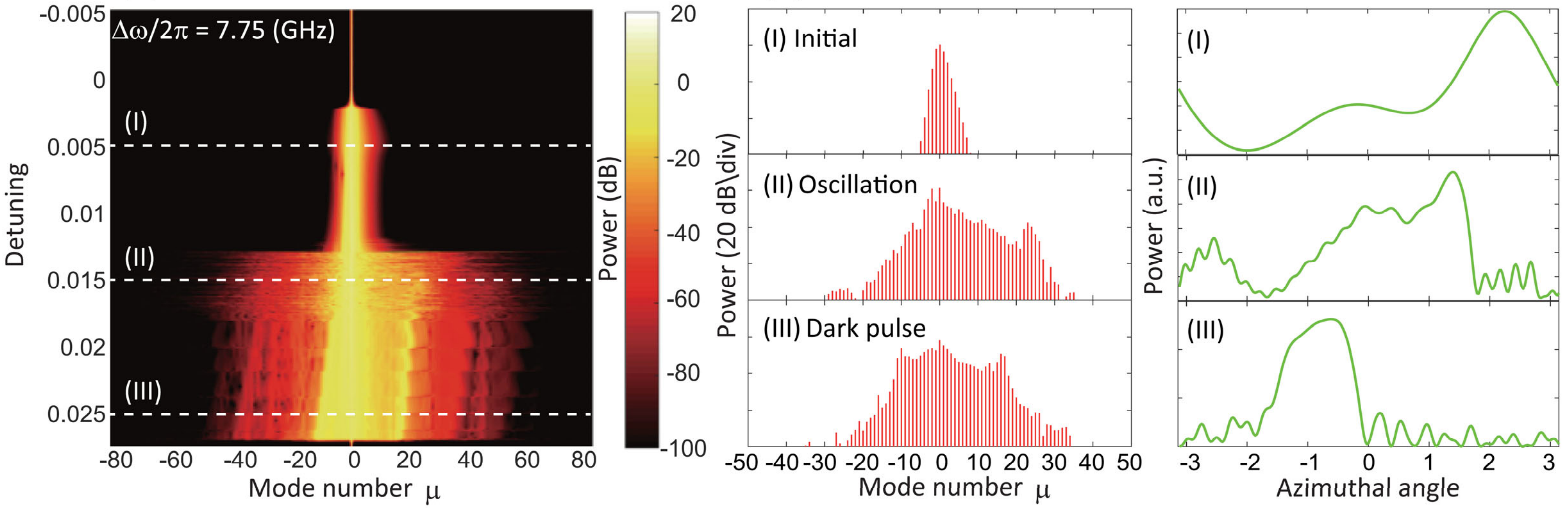
Perfect agreement w/ experiments performed by:
A. Weiner (Purdue Univ.) X. Xue, et al. Nat. Photonics 9, 594 (2015), Laser Photon. Rev. 9, L23 (2015).

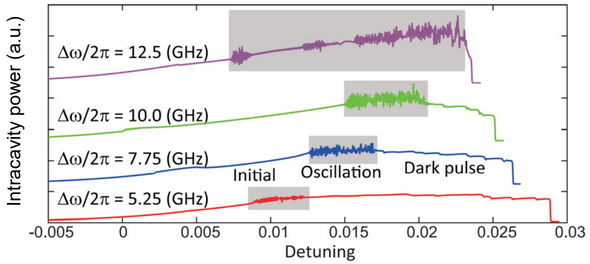
Observation of noisy state

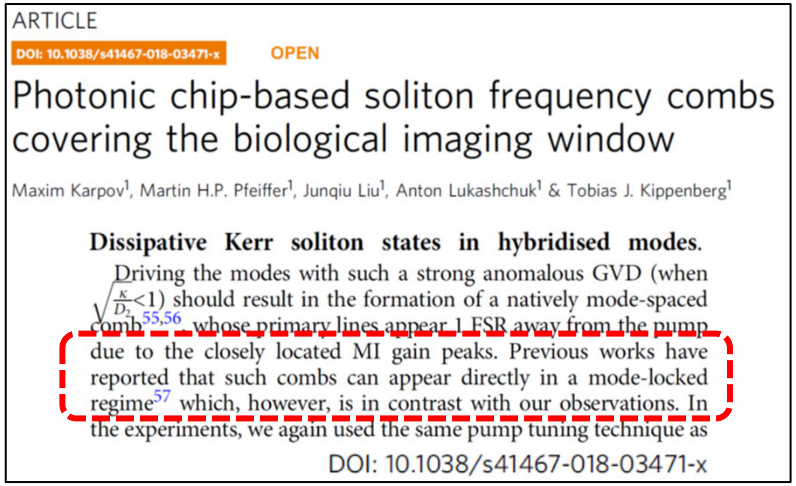
▶Direct transition to ML



▶Observation of noisy state (in normal dispersion)






ARTICLE

DOI: 10.1038/s41467-018-03471-x OPEN

Photonic chip-based soliton frequency combs covering the biological imaging window

Maxim Karpov[1], Martin H.P. Pfeiffer[1], Junqiu Liu[1], Anton Lukashchuk[1] & Tobias J. Kippenberg[1]

Dissipative Kerr soliton states in hybridised modes. Driving the modes with such a strong anomalous GVD (when $\sqrt{\frac{\kappa}{D_2}}<1$) should result in the formation of a natively mode-spaced comb[55,56] whose primary lines appear 1 FSR away from the pump due to the closely located MI gain peaks. Previous works have reported that such combs can appear directly in a mode-locked regime[57] which, however, is in contrast with our observations. In the experiments, we again used the same pump tuning technique as

DOI: 10.1038/s41467-018-03471-x

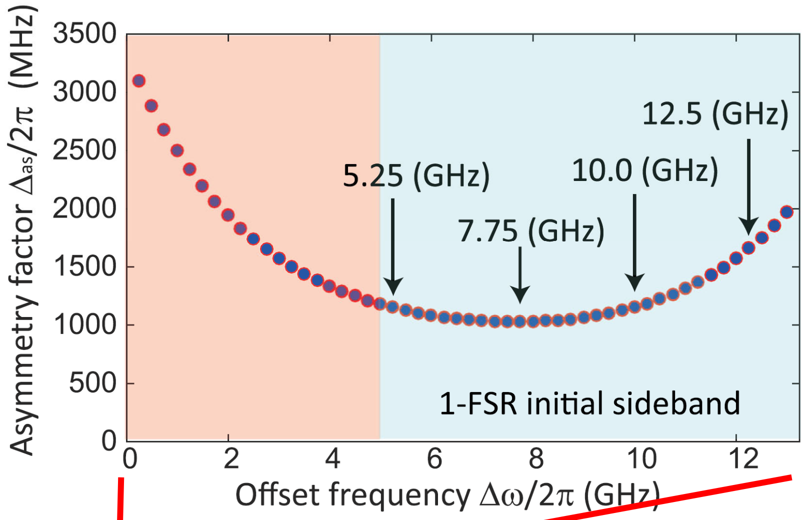
Noisy state is present when generating 1-FSR comb

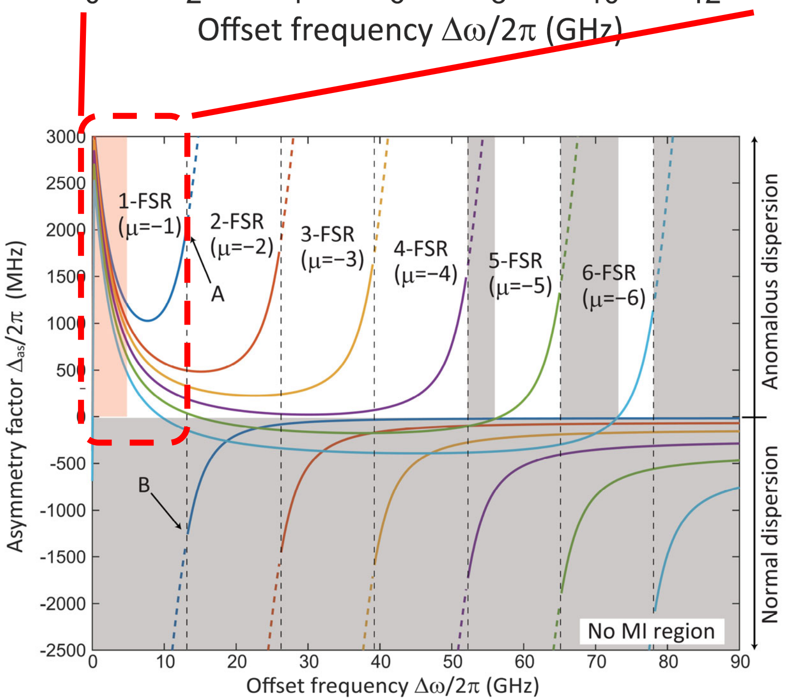
Questions & discussions raised

- ✓ Why do we observe noisy state in normal dispersion system?
- ✓ Normal dispersion system was supposed to reach directly in a mode-locked regime

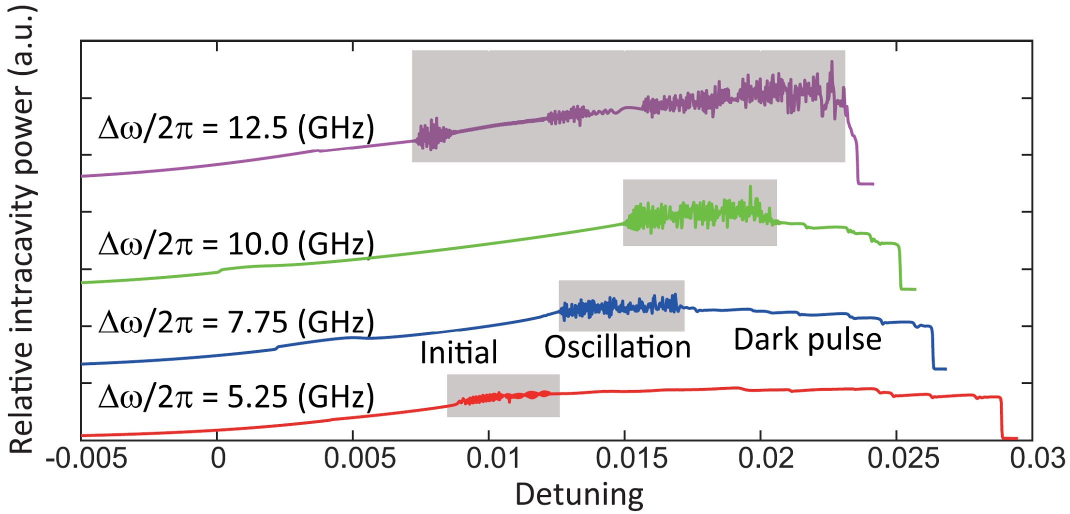
Deterministic FSR generation is not always possible

▶Phase matching condition





▶Oscillation behavior



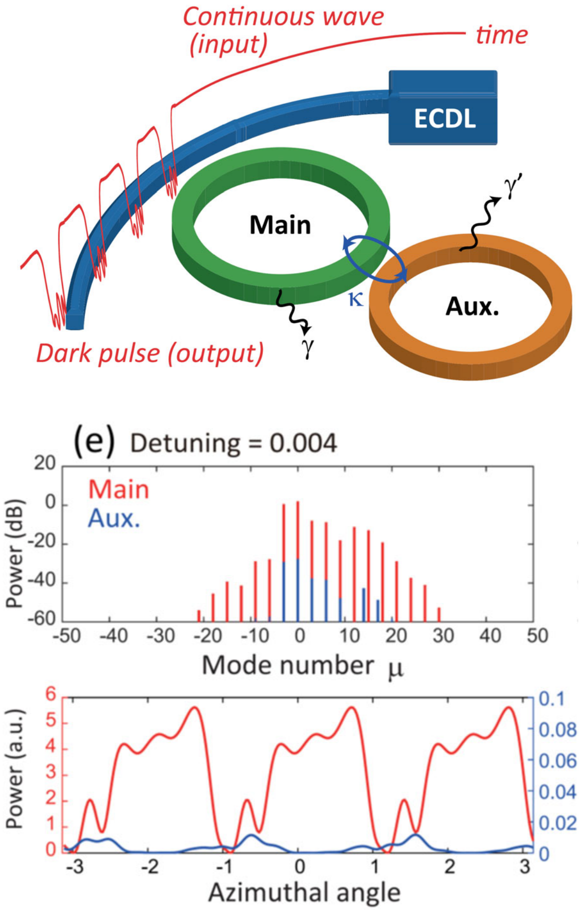
✓ Offset frequency is at the blue shaded region:
- 1-FSR comb generates.
- Agrees with the observation by [Y. Liu, Optica 1 137 (2014)]

✓ Offset frequency is a the red shaded region:
- strong oscillation is observed
- Agrees with the observation by [M. Karpov, Nat. Commun., 9, 1146 (2018)]

We found that deterministic FSR generation in a normal dispersion system is not always possible even when using coupled cavities.

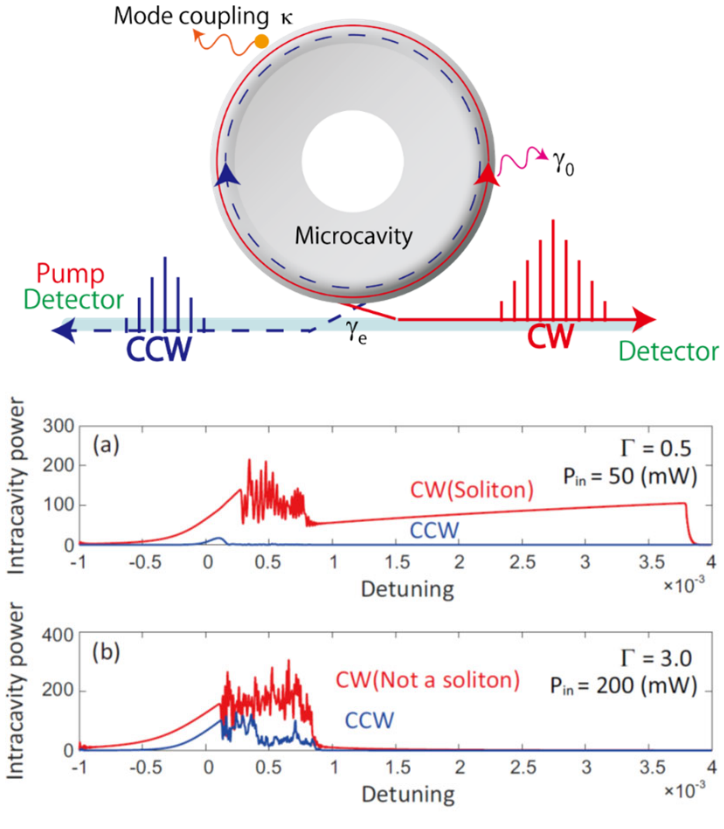
Kerr combs with mode coupling

Two modes coupling



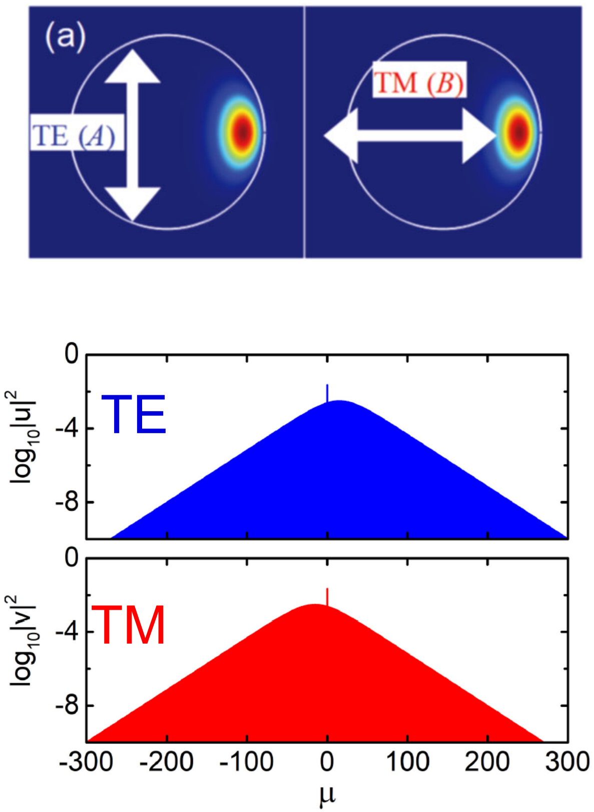
S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).

CW/CCW mode coupling



S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

TE/TM mode coupling



R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

Dark pulse generation at normal dispersion

Effect of inherent coupling

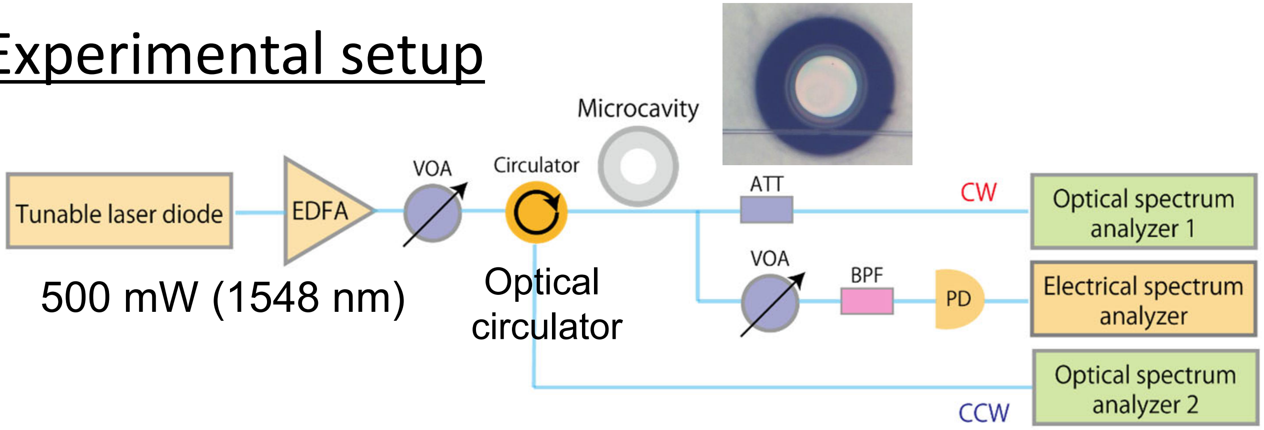
Dual comb generation

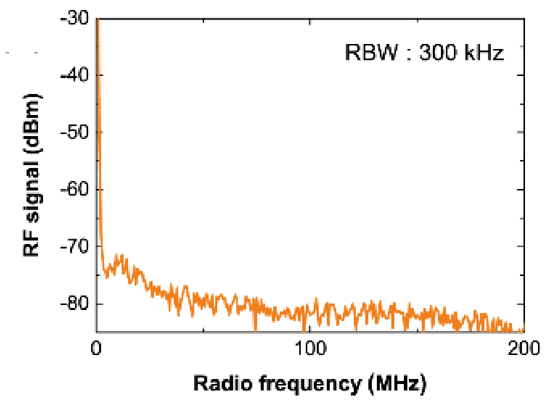
CW-CCW comb measurement

Silica toroid ($Q = 2 \times 10^7$)

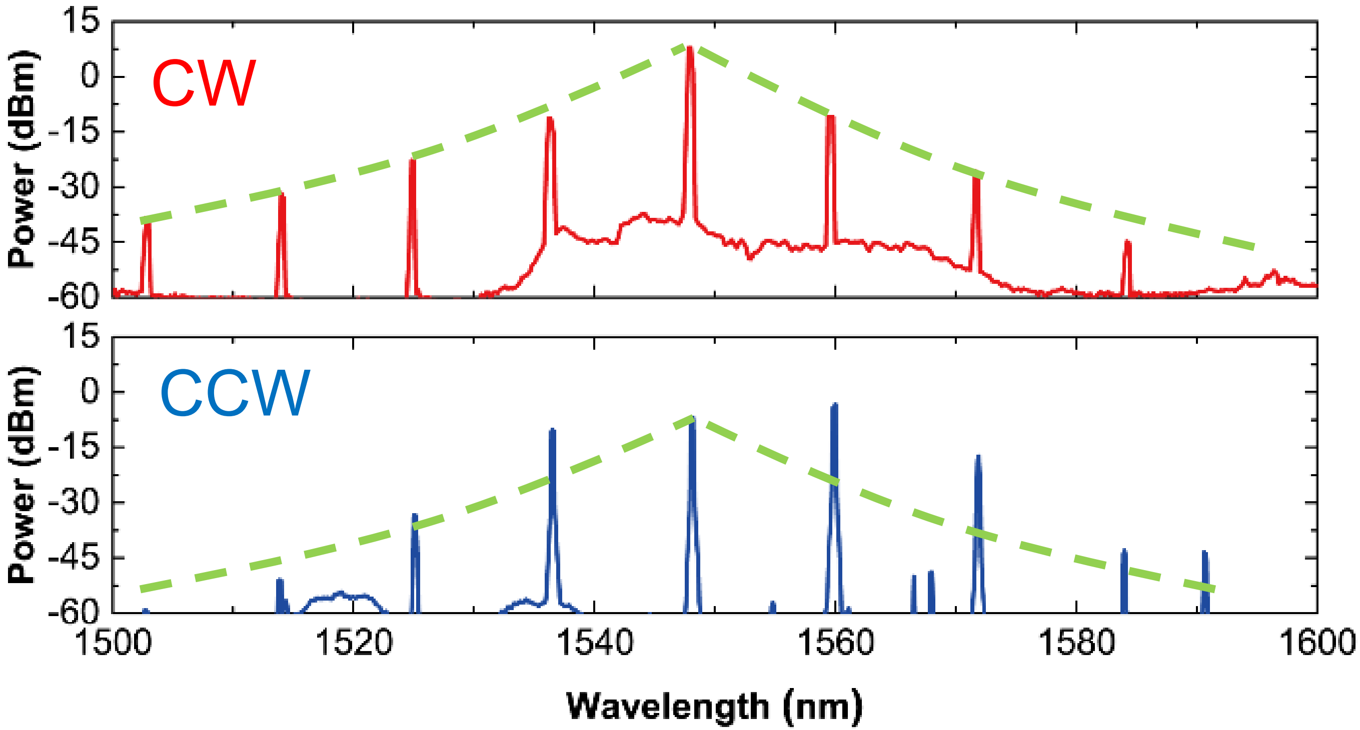
S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

Experimental setup





Results



CW direction

Triangular envelope

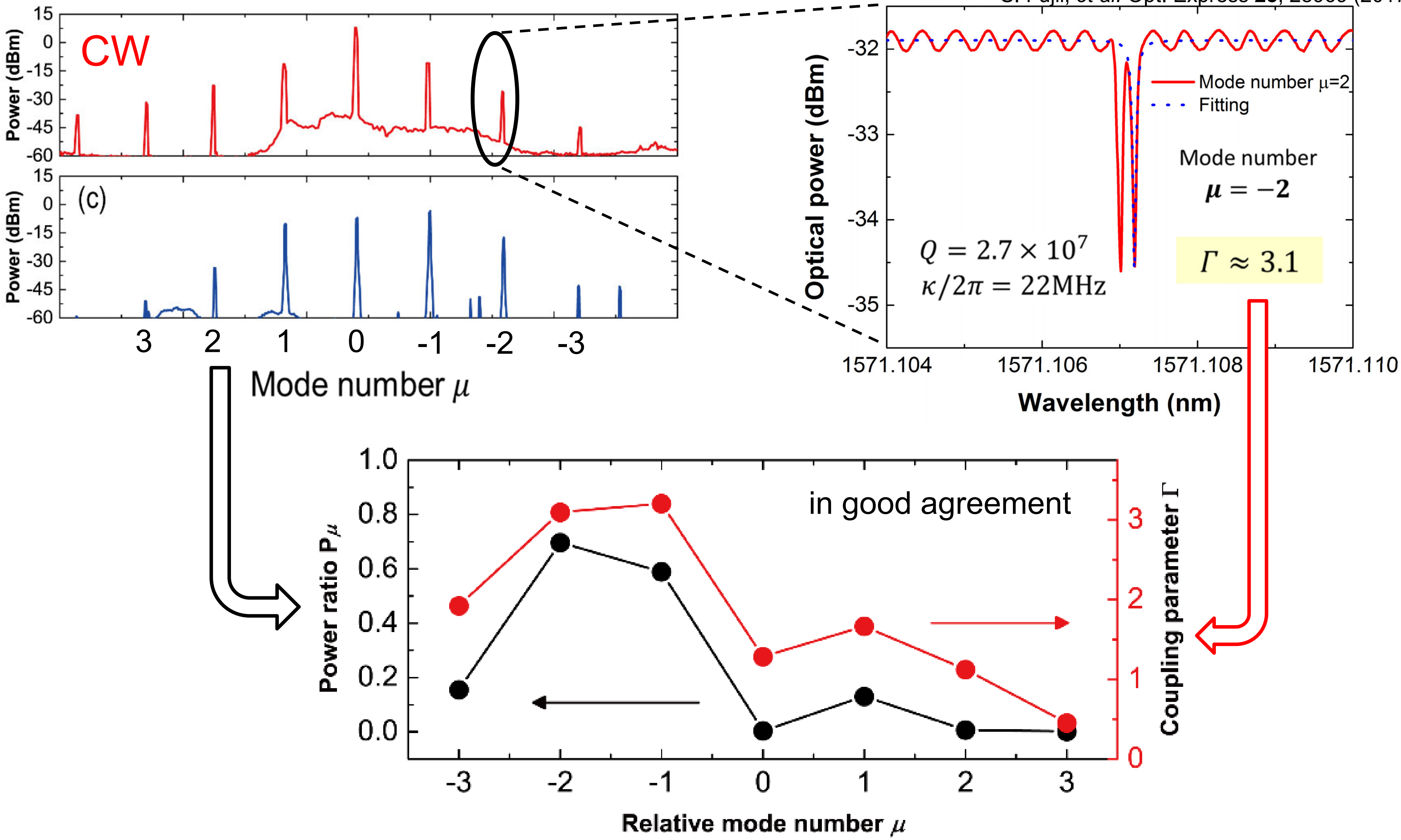
Phase-locked ?

CCW direction

Envelope is not smooth

Effect of CW/CCW coupling (experimental)

S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).



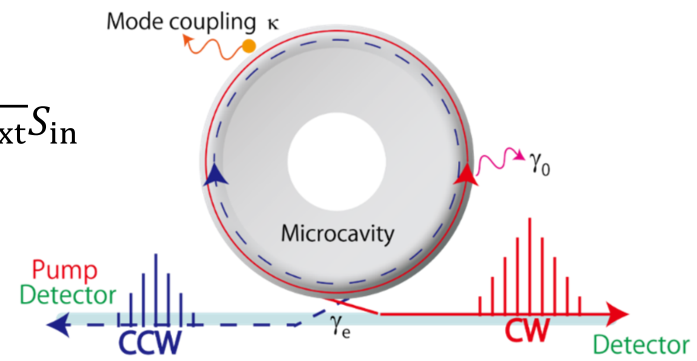
Effect of CW/CCW coupling (numerical)

▶Model

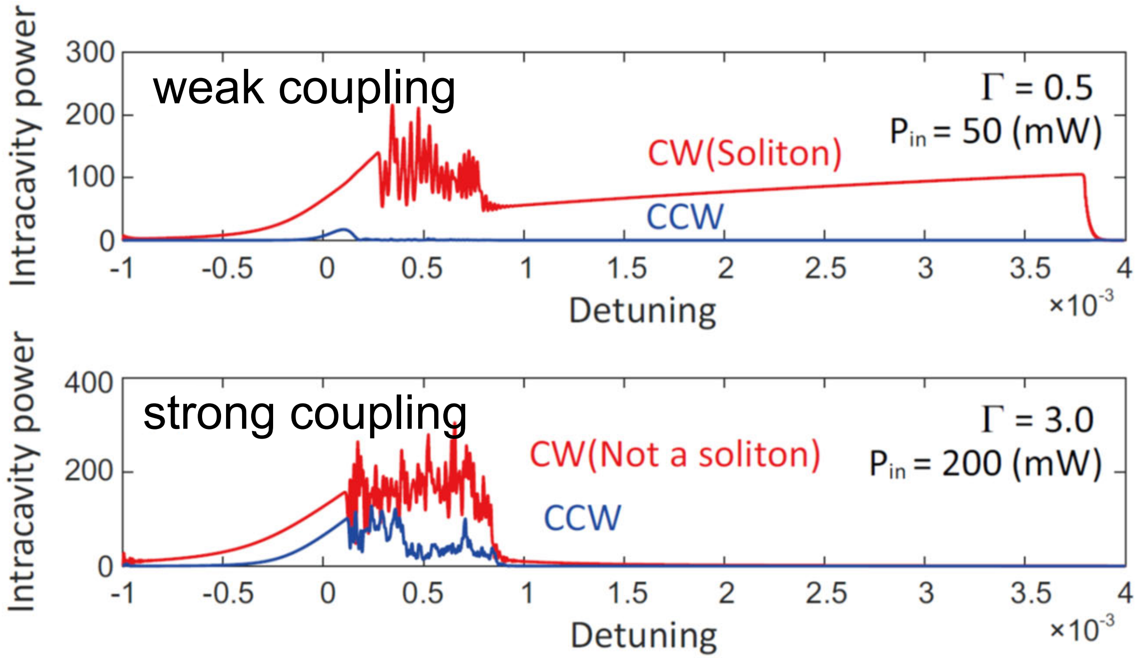
S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

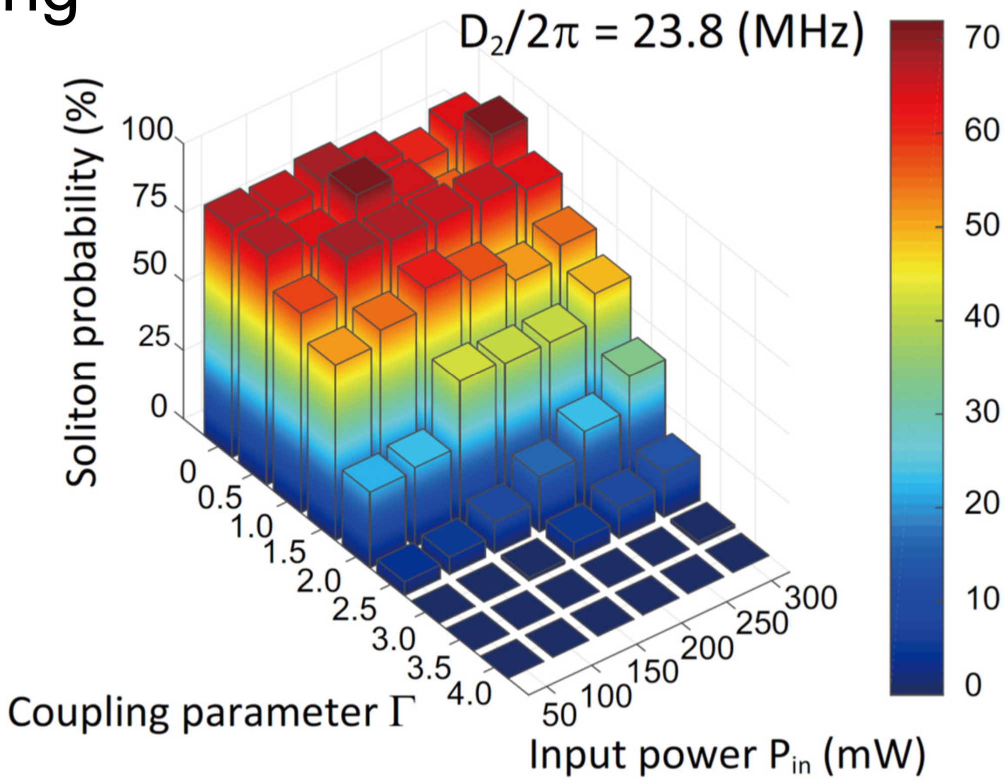
$$\frac{\partial A(\phi,t)}{\partial t} = -\left(\frac{\gamma}{2} + i\delta_0\right) A + i\frac{D_2}{2}\frac{\partial^2 A}{\partial \phi^2} + ig\left(|A|^2 + 2|B|^2\right)A + i\frac{\kappa_\mu}{2}B + \sqrt{\gamma_{\text{ext}}}S_{\text{in}}$$

$$\frac{\partial B(\phi',t)}{\partial t} = -\left(\frac{\gamma}{2} + i\delta_0\right) B + i\frac{D_2}{2}\frac{\partial^2 B}{\partial \phi'^2} + ig\left(|B|^2 + 2|A|^2\right)B + i\frac{\kappa_\mu}{2}A$$



▶Soliton formation w/ different coupling

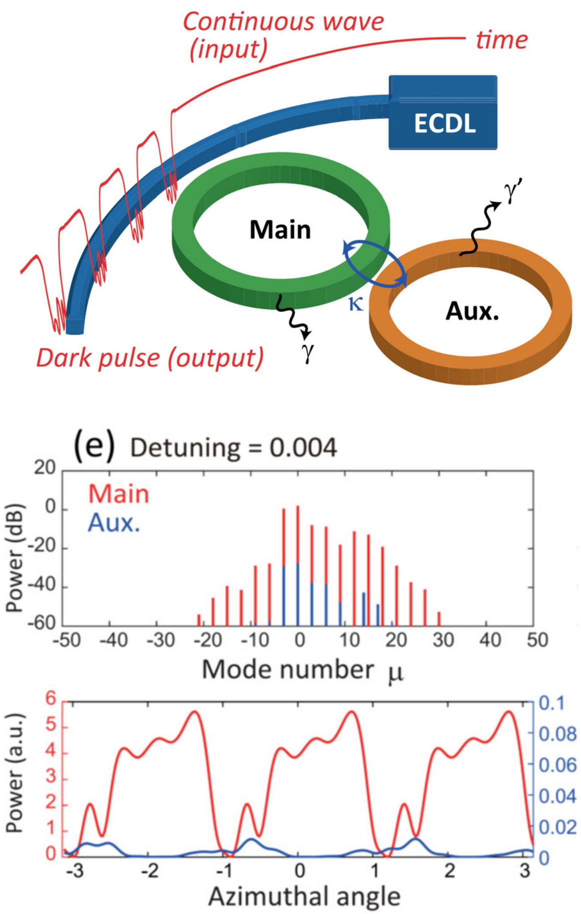




Effect of coupling is present but, usually it is negligible

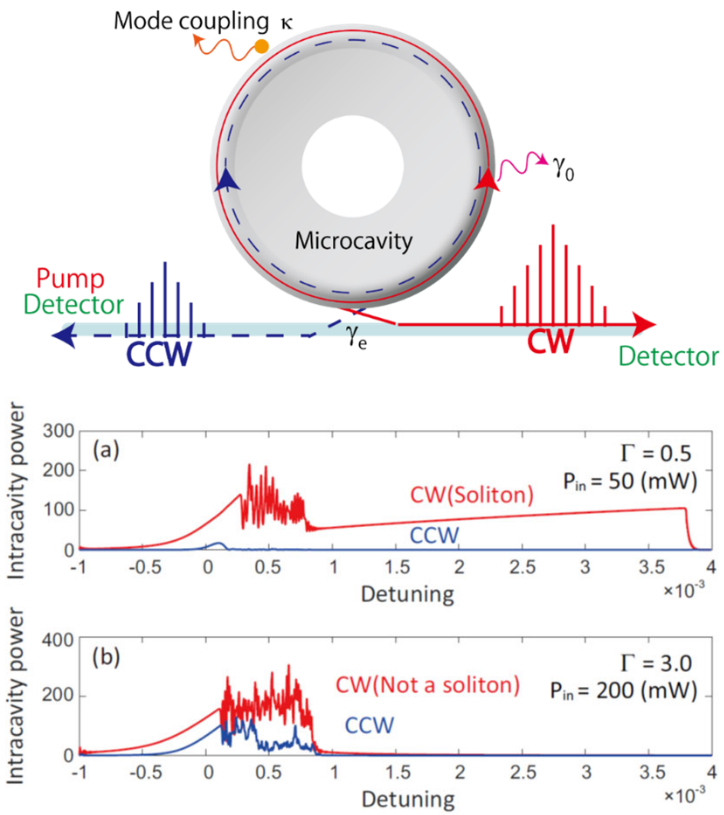
Kerr combs with mode coupling

Two modes coupling



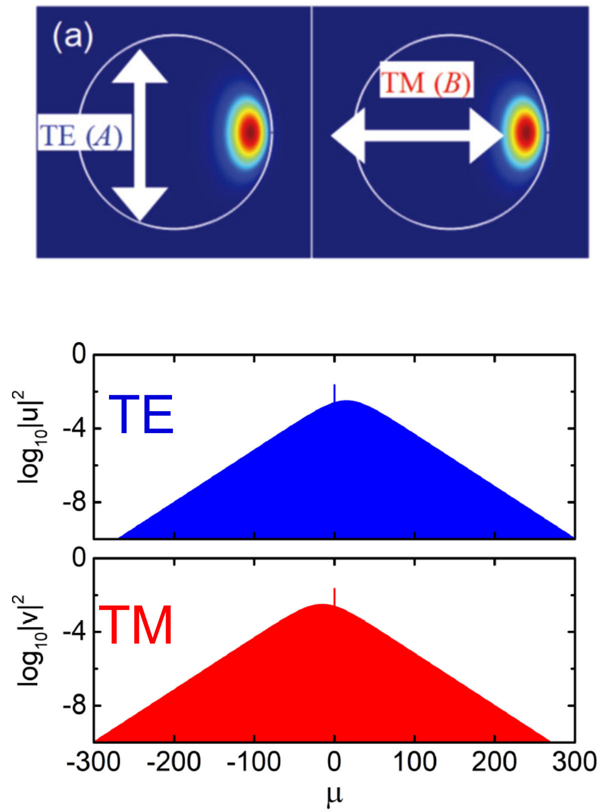
S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).

CW/CCW mode coupling



S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

TE/TM mode coupling



R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

Dark pulse generation at normal dispersion

Effect of inherent coupling

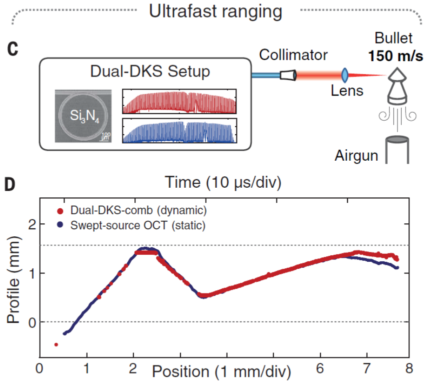
Dual comb generation

Dual-comb applications

Dual-comb applications: scan rate ⇔ difference of repetition frequencies

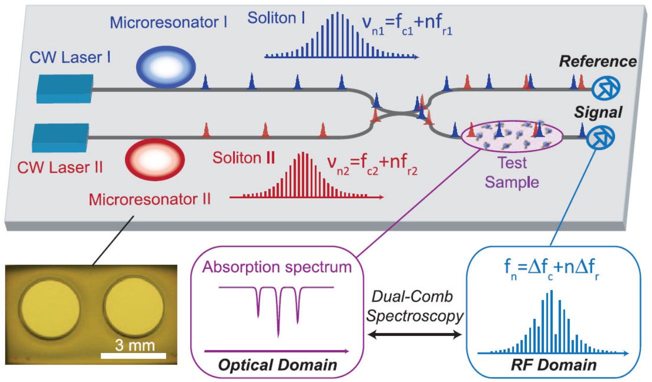
Microcombs have a potential to achieve fast scan rate due to high repetition frequencies

LiDAR



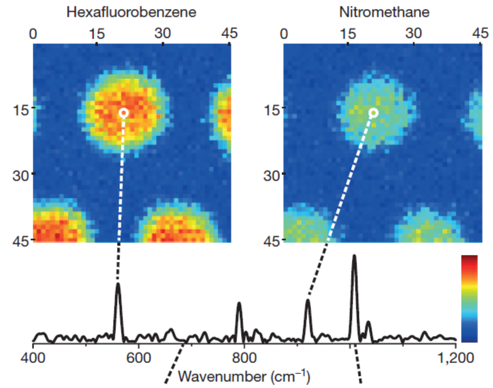
Science 359, 887-891 (2018)

Spectroscopy



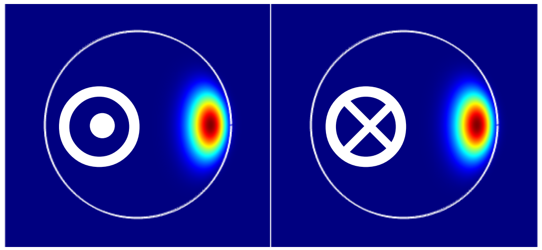
Science 354, 600-603 (2016)

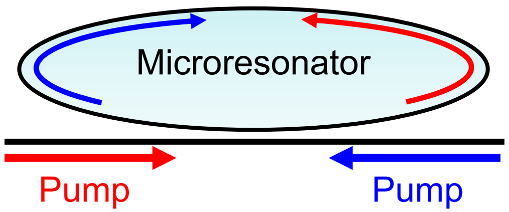
CARS



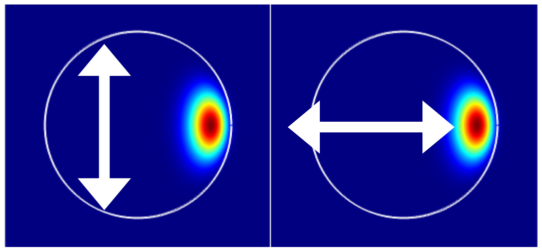
Nature 502, 355-358 (2013)

CW/CCW directions

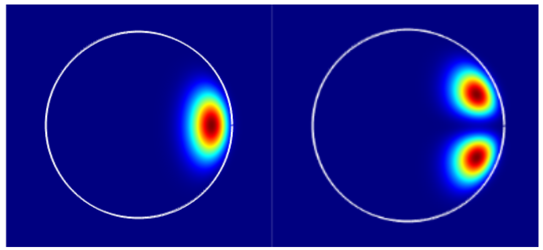


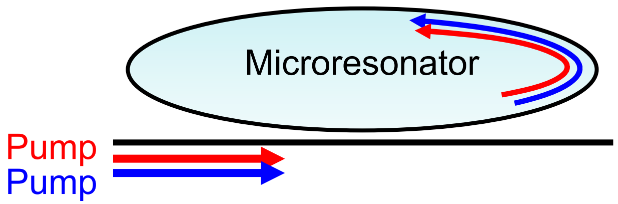


TE/TM modes



Transverse modes





☺ Simple control of pump frequencies

☹ Small repetition rate difference

☹ Complex control of pump frequencies

☺ Large repetition rate difference

TE/TM mode coupling (numerical model)

Coupled Lugiato-Lefever equations (LLEs)

R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

$$\frac{\partial a}{\partial t} = -\frac{\kappa_{(a)}}{2}a + i\Delta\omega_{0(a)}a + i\frac{D_{2(a)}}{2}\frac{\partial^2 a}{\partial \phi^2} + ig_{(a)}(|a|^2 + \sigma|b|^2)a + \sqrt{\kappa_{c(a)}}s_{\mathrm{in}(a)} + \frac{\Delta D_1}{2}\frac{\partial a}{\partial \phi}$$

$$\frac{\partial b}{\partial t} = -\frac{\kappa_{(b)}}{2}b + i\Delta\omega_{0(b)}b + i\frac{D_{2(b)}}{2}\frac{\partial^2 b}{\partial \phi^2} + ig_{(b)}(|b|^2 + \sigma|a|^2)b + \sqrt{\kappa_{c(b)}}s_{\mathrm{in}(b)} - \frac{\Delta D_1}{2}\frac{\partial b}{\partial \phi}$$

(loss) (detuning) (dispersion) (Kerr effects) (input) (repetition difference)

t: time, ϕ: angular coordinate, a, b: internal fields, κ: resonator loss, $\Delta\omega_0$: pump detuning, D_2: second order dispersion, g: nonlinear coefficient, σ: XPM coefficient (σ = 2/3 for orthogonally polarizations), κ_c: coupling rate, s_{in}: input field, ΔD_1: FSR (repetition frequency) difference

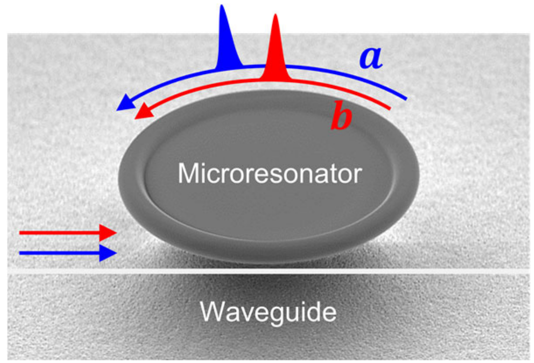
Dimensionless coupled LLEs

(Assuming $\kappa = \kappa_{(a)} = \kappa_{(b)}$, $g = g_{(a)} = g_{(b)}$)

$$\frac{\partial u}{\partial \tau} = -(1 + i\alpha_{(u)})u + i\beta_{(u)}\frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma|v|^2)u + F_{(u)} + \gamma\frac{\partial u}{\partial \phi}$$

$$\frac{\partial v}{\partial \tau} = -(1 + i\alpha_{(v)})v + i\beta_{(v)}\frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + \sigma|u|^2)v + F_{(v)} - \gamma\frac{\partial v}{\partial \phi}$$

$$\tau = \frac{1}{2}\kappa t,\ u = \sqrt{\frac{2g}{\kappa}}a,\ v = \sqrt{\frac{2g}{\kappa}}b,\ \alpha_{(*)} = -\frac{2\Delta\omega_{0(*)}}{\kappa},\ \beta_{(*)} = \frac{D_{2(*)}}{\kappa},\ \gamma = \frac{\Delta D_1}{\kappa},\ F_{(*)} = \frac{2}{\kappa}\sqrt{\frac{2g\kappa_{c(*)}}{\kappa}}s_{\mathrm{in}(*)}$$



Relations, α: detuning, β: second order dispersion, γ: repetition difference, F: input

Soliton trapping with dimensionless coupled LLEs

$$\frac{\partial u}{\partial \tau} = -(1 + i\alpha_{(u)})u + i\beta_{(u)}\frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma|v|^2)u + F_{(u)} + \gamma\frac{\partial u}{\partial \phi}$$

$$\frac{\partial v}{\partial \tau} = -(1 + i\alpha_{(v)})v + i\beta_{(v)}\frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + \sigma|u|^2)v + F_{(v)} - \gamma\frac{\partial v}{\partial \phi}$$

R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

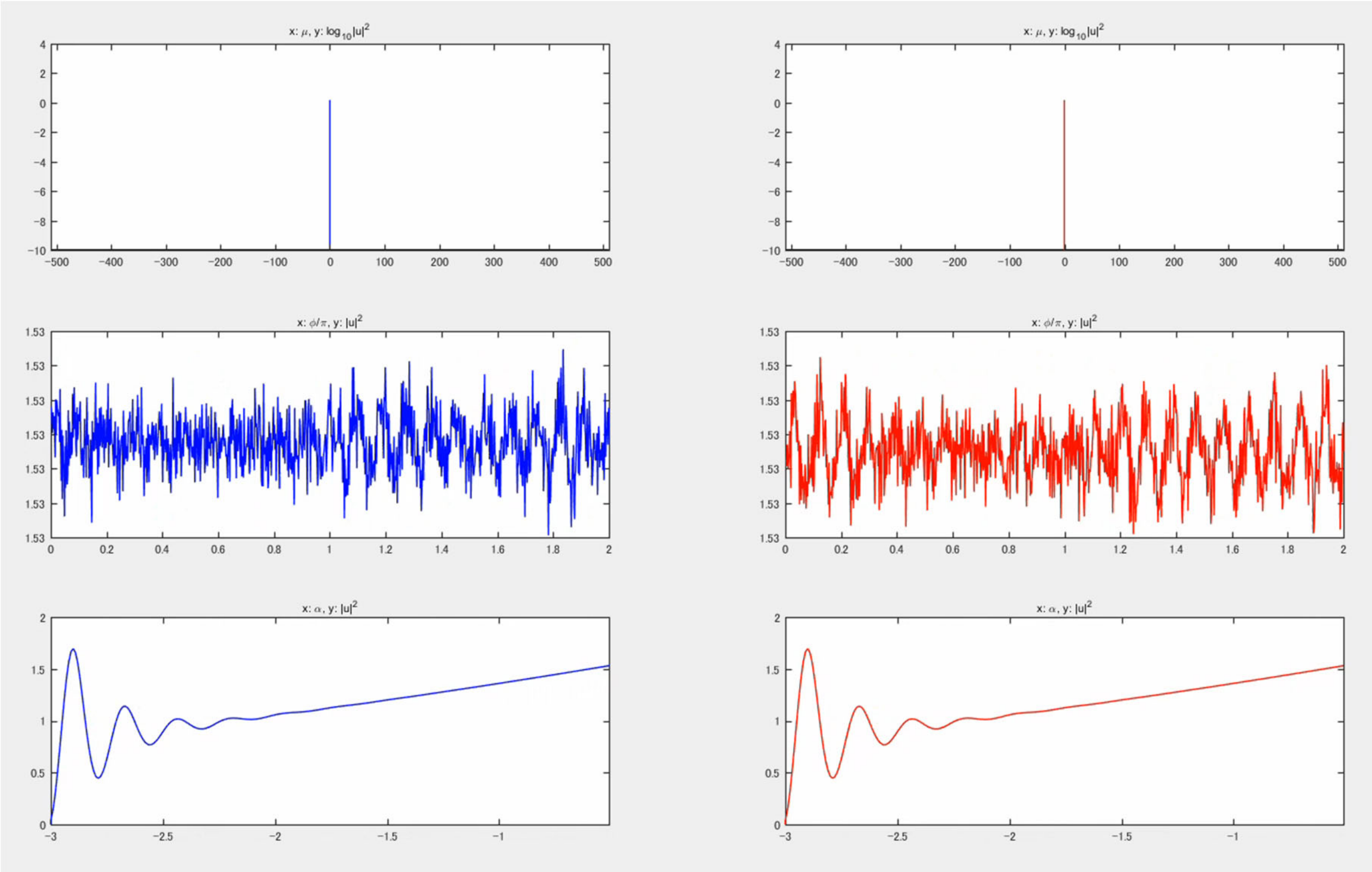
$\beta_{(*)} = 0.01, \gamma = 0.3, F_{(*)} = 4$
α is scanned

Spectrum

Waveform

Moving at different speeds:
Micorocombs propagate
at different group velocities

Intracavity
power



α: detuning, β: second order dispersion, γ: repetition difference, F: input

Soliton trapping with dimensionless coupled LLEs

$$\frac{\partial u}{\partial \tau} = -(1 + i\alpha_{(u)})u + i\beta_{(u)}\frac{\partial^2 u}{\partial \phi^2} + i(|u|^2 + \sigma|v|^2)u + F_{(u)} + \gamma\frac{\partial u}{\partial \phi}$$

$$\frac{\partial v}{\partial \tau} = -(1 + i\alpha_{(v)})v + i\beta_{(v)}\frac{\partial^2 v}{\partial \phi^2} + i(|v|^2 + \sigma|u|^2)v + F_{(v)} - \gamma\frac{\partial v}{\partial \phi}$$

R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

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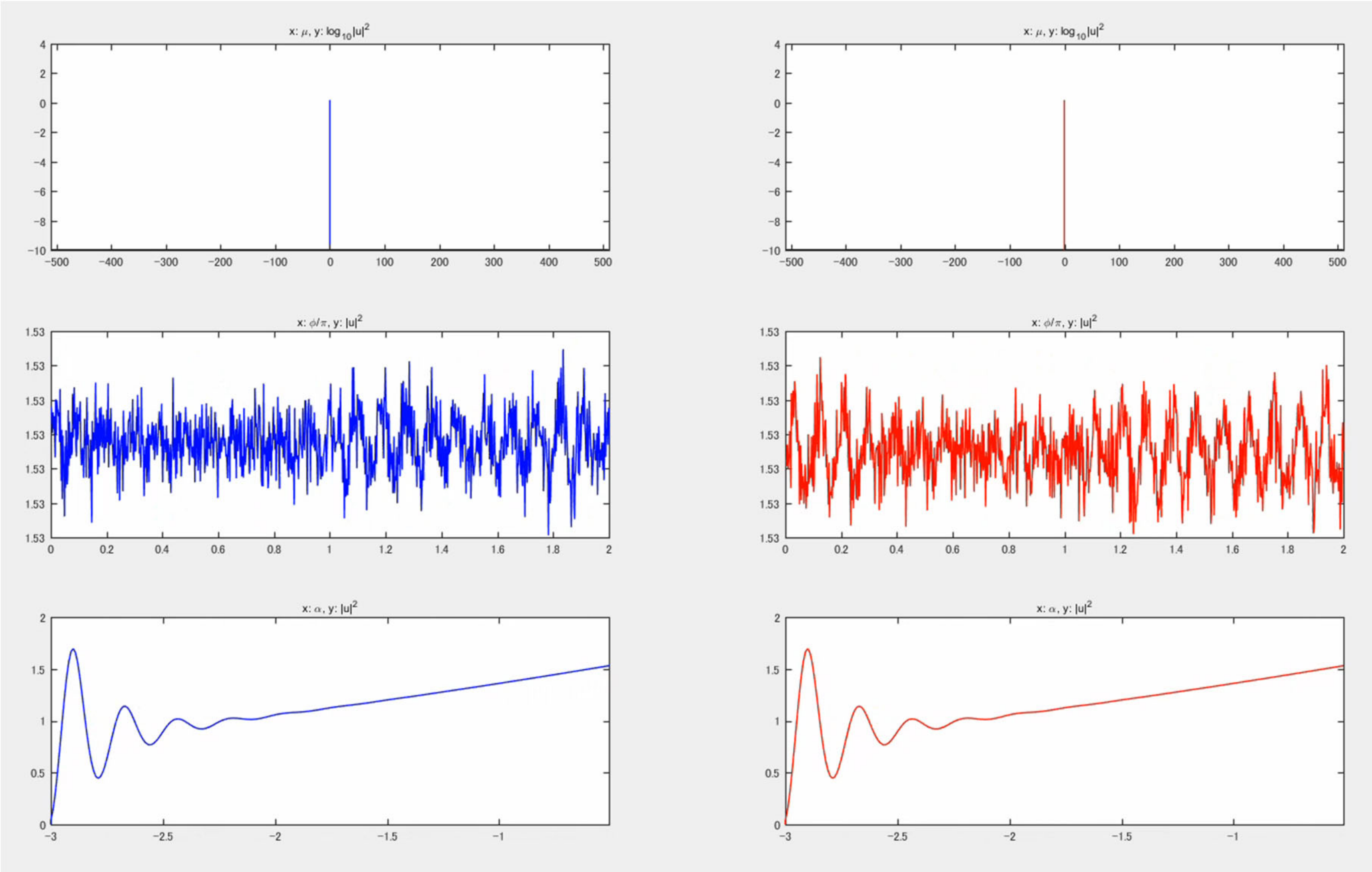
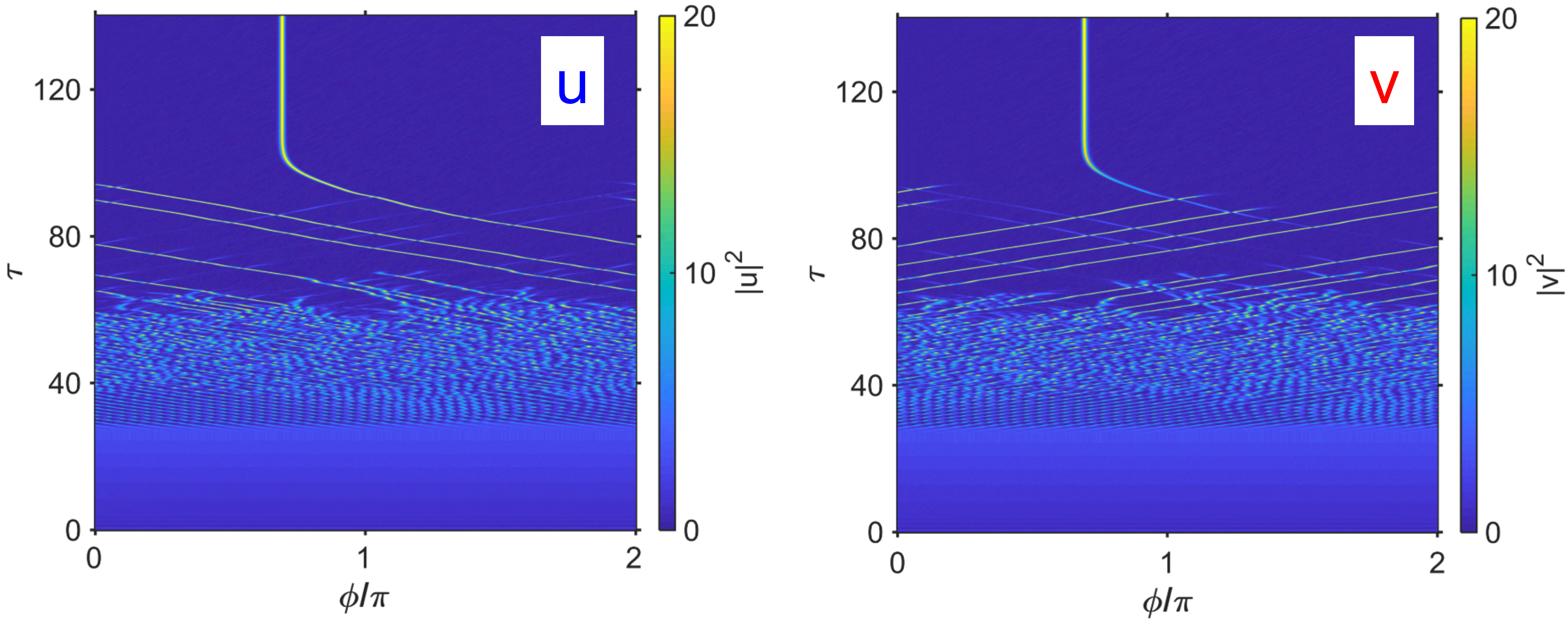


α: detuning, β: second order dispersion, γ: repetition difference, F: input

Soliton build-up

R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

▶Waveforms



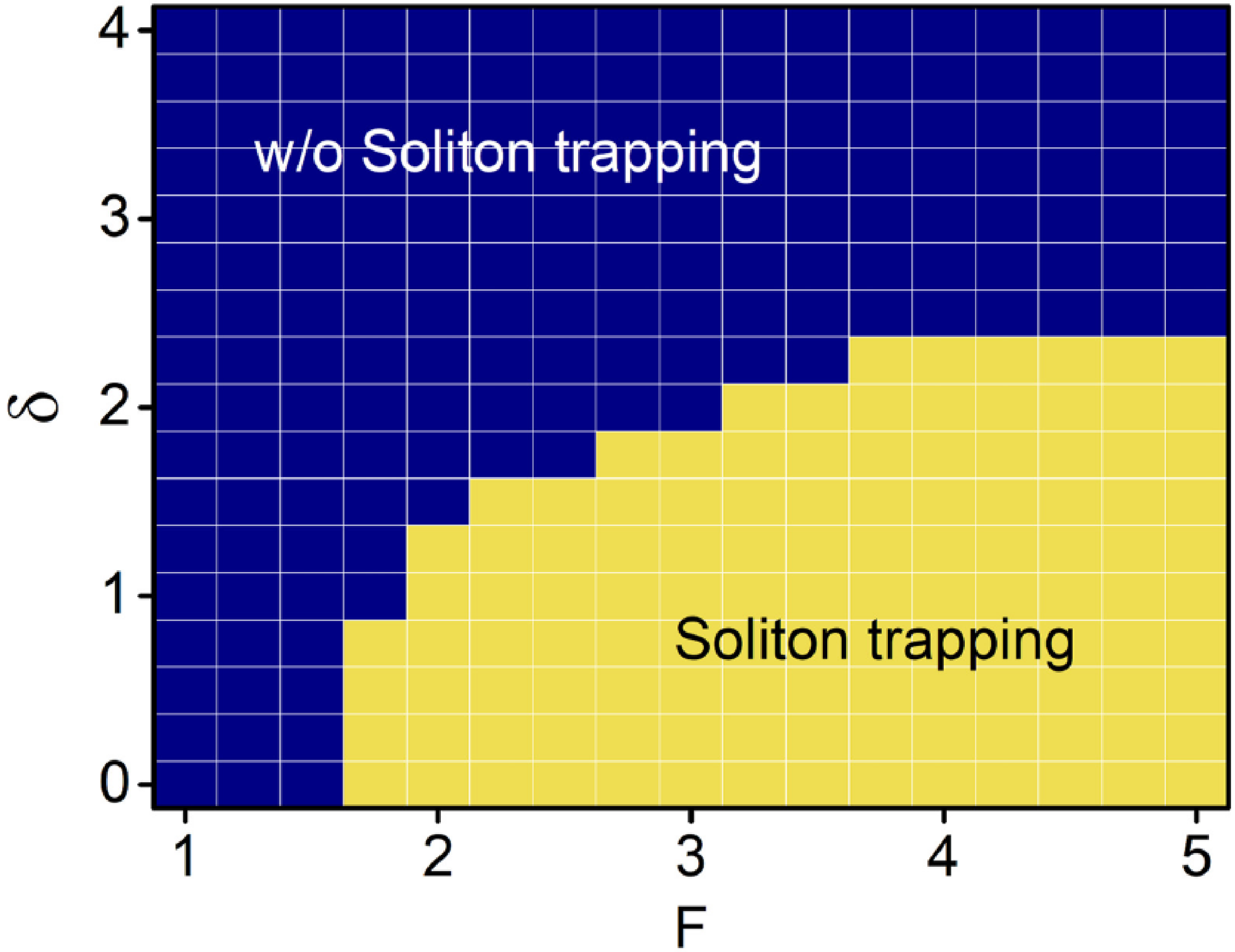
α: detuning, β: second order dispersion, γ: repetition difference, F: input, $\delta = \gamma(2\beta)^{-0.5}$

Trapping conditions as functions of F and δ

▶Trapping conditions

R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

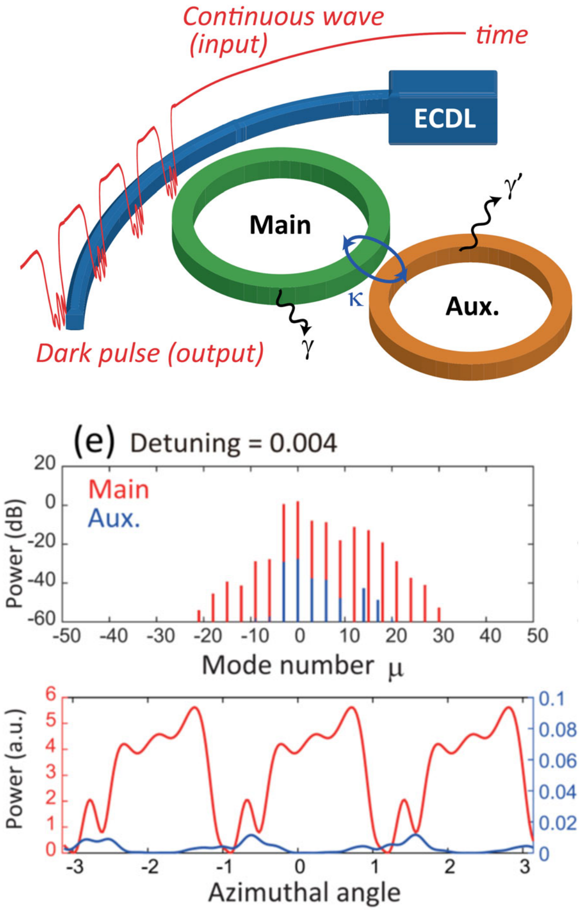
(as functions of F (input) and δ (rep. difference))



Relations α: detuning, β: second order dispersion, γ: repetition difference, F: input, $\delta = \gamma(2\beta)^{-0.5}$

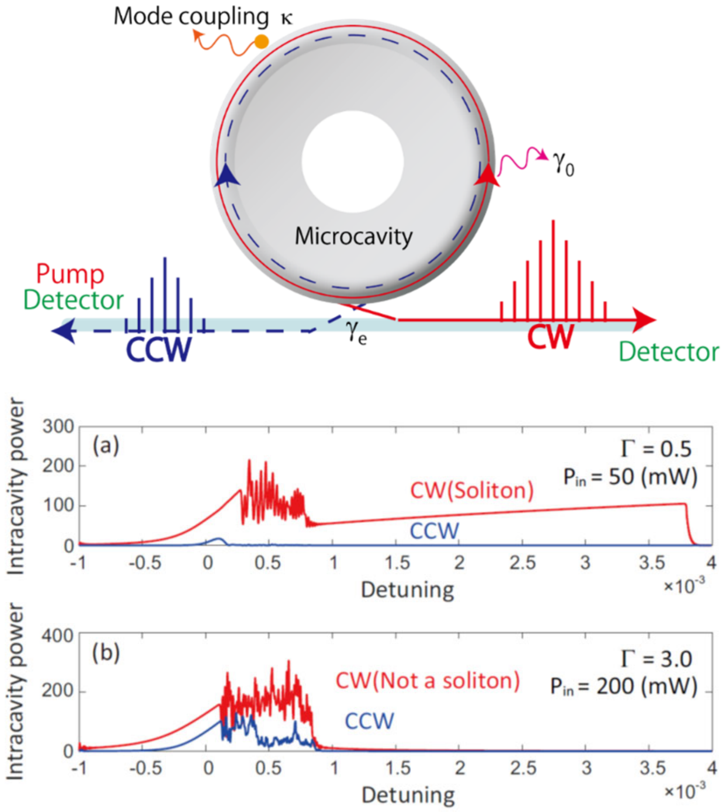
Summary

Two modes coupling



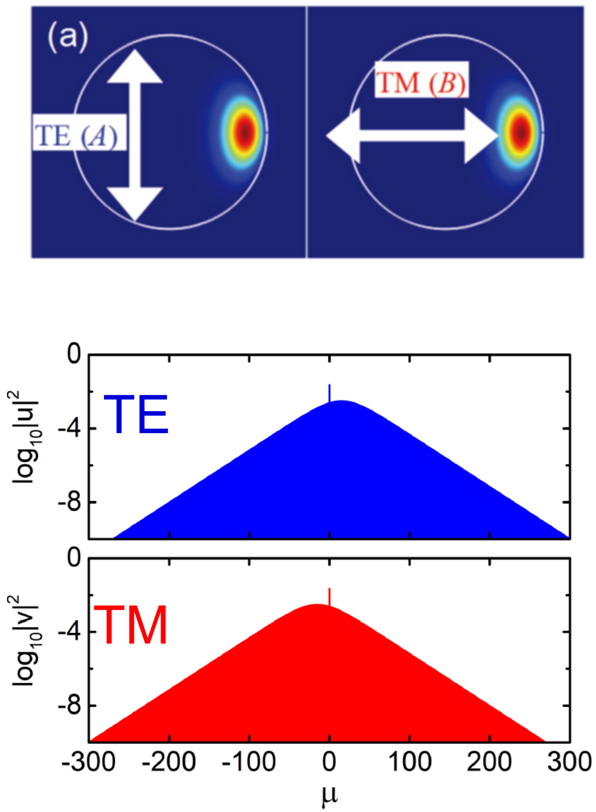
S. Fujii, *et al.* IEEE Phot. J., **10**, 4501511 (2018).

CW/CCW mode coupling



S. Fujii, *et al.* Opt. Express **25**, 28969 (2017).

TE/TM mode coupling



R. Suzuki, *et al.* IEEE Phot. J. **11**, 6100511 (2019).

Dark pulse generation at normal dispersion

Effect of inherent coupling

Dual comb generation

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- Support

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